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Enforcement and equilibrium in the permit markets when firms are risk averse

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by

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ABSTRACT

This paper explores the role of uncertainty, in the form of measurement error, in pollution regulation under a tradable permit system. In particular, we showed the neutrality between the penalty and the audit frequency does not hold when agents (firms) are risk averse. Firms respond to the weight change between penalty and monitoring effort by adjusting their demand for pollution permits, as well as their production/pollution decisions. We studied two forms of the measurement error when observing the emissions: additive and multiplicative. While there are some analytical results for a model with additive error, the same cannot be said when the error is multiplicative to the real emission. We then used numerical methods to simulate firms behavior and the industry equilibrium with multiplicative error, and to identify the best policy for the government.

Keywords: *risk, permits, pollution, penalty, market equilibrium, free-entry*

CHAPTER 1: INTRODUCTION

Pollution permit markets long have been of interest for environmentalists and economists in promoting better environmental quality. It is recognized that, under normal circumstances, tradable permits are the most efficient way of pollution control in terms of cost minimization. There is a large   literature on permit markets since its emergence in the 80's. While economists generally agree that permit markets are more efficient than the traditional bureaucratic "command-and-control" method for improving environmental quality, there are other aspects of it that have attracted heated discussion among scholars over the last three decades. Among them, one major debate is about the effectiveness of the environmental standard enforcement. A handful of researchers have studied the enforcement issue in permit markets, under different assumptions about the market's nature. Malik (1990, 2002) has assessed the permit market equilibrium and the eventual pollution level when firms could be noncompliant to the environmental regulations, with the uncertainty of being audited. Keeler (1991) extends Malik's basic model by looking into the effects of different penalty functions on firms' compliance decisions and thus, impacts on the total pollution level. Later, Standlund and Dhanda (1999) considered a model in which firms have different exogenous characteristics that may effect their abatement costs; in their model they argued that these exogenous factors should not effect firms' compliance decisions, and, as such, regulators need not distinguish among firms to allocate enforcement resources based on their exogenous characteristics.[1] A paper by Mrozek and Keeler (2004) built a 2-period model in which the

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[1] The exogenous factors would effect equilibrium permit prices, through their impacts on abatement cost functions.

firms' emissions were stochastic, in order to study the effect of emission uncertainty on the equilibrium permit prices[2].

Those papers addressed the issue of imperfect enforcement in permit markets assuming firms were risk-neutral, and there was no discussion on the limitation of fines[3]. This risk-neutral assumption made the analyses simpler, but it is less plausible in a real world scenario, especially in the case when the measured emissions might be stochastic either because of monitoring deficiency, or, as in Mrozek and Keeler's paper, due to the output uncertainty in the product markets. Ben-David *et al.* (2000) also suggested the uncertainty may come from permit markets' fluctuation; they postulated that, when permit price is uncertain, risk averse permit buyers would demand fewer permits and abate more pollution accordingly, whereas risk averse permit sellers did the opposite. Stranlund (2008) later assessed the role of firms' risk attitude on the permit market, where uncertainty solely came from the random inspections performed by the regulators. They concluded that the risk attitude would not affect firms' compliance decision, much as in Malik's paper, though it did impact the firms' behavior with respect to the number of permits to hold, once they have decided to be compliant or not. The paper by Ozanne and White (2008) offers another study of risk attitude on environmental regulation. They examine the interaction of farmer's risk attitude and government's input-based regulation via contract theory. The work of Bontems and Rotillon

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[2] They focused mainly on the comparison of equilibrium permit prices and the marginal penalty in both periods, yet the setup of the model lacked a clear definition about the prices in different periods; the credibility of their results are a bit daunting.

[3] In the paper by Stanlund and Dhanda they set a budget constraint on the government's expense, but still there's no reasoning for an upper bound of fines in their model.

(2007) added the twist of "involuntary violation" due to some stochastic factor[4], into a model of environmental standard with risk averse agents; they assumed there is some "social norm" that binds agents behavior, and discuss the relevant compliance decision of the agents at the equilibrium.

While there are some papers that study the impact of risk attitude on the permit markets, the construction of fines, nonetheless, has yet been little explored. In the criminal economics literature, it has been long argued that the most efficient way to deter crimes is to set audit/ enforcement probability to its minimal level and let the fine go to its maximum level, e.g., set it to the violator's wealth.[5] Yet this statement has been challenged a lot in later and recent research, for there are many conditions under which it does not hold.[6] Polinsky and Shavell (1979) first pointed out that with risk averse agents, the optimal monitoring probability would not necessarily be zero, nor the corresponding fine to be set to its maximum. A more recent paper by Arguedas (2008) further assessed the relationship between an endogenous enforcement policy and firms compliance behavior, under a environmental standard system.

In light of previous research, we will expand the literature on pollution permits by assessing how two major factors, namely firms' risk attitude and the penalty structure, interplay with each other in a competitive permit market. While previous research has studied these two factors separately, there has not yet been a model that integrates them. Moreover, to fully understand the impact of risk attitude on permit market, it is necessary to identify the

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[4] It was not clearly defined where this "involuntary violation" came from in the model.

[5] See Becker (1968).

[6] See Cameron (1988).

uncertainty in the model. On this point, we construct a model with "measurement uncertainty," in which the government may erroneously observe a firm's emission.[7]

This dissertation is structured as the following: Chapter 1 is a review of the relevant literature; in Chapter 2 we construct a formal model addressing additive measurement error. Then in Chapter 3, we advance the additive error model to multiplicative error, reflecting the market condition more realistically. Chapter 4 we discuss the government's policy instruments and the industry equilibrium. Chapter 5 is the simulation results for the multiplicative error case. A conclusion is presented lastly.

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[7] From an analytic perspective, it is the same structure to assume either the error is from the regulator's measurement, or that the firm's emission itself may be stochastic.

CHAPTER 2: LITERATURE REVIEW

1. Permit Price Uncertainty and Risk-averse Firms

In the paper by Ben-David et al. (2000), they assessed the impact of permit price uncertainty on firms' emission and permit-buying decision, by constructing a rather straightforward model, testing the hypothesis that, under uncertainty, the ex-ante equilibrium permit price would be the same, while the ex-ante trading volume decreases, and the ex-ante efficiency is lower (in the sense of cost-saving feature for a permit scheme.)[8] To understand their work, let q be the abated discharge amount. Firms differ in the marginal abatement cost function, $a_i(q_i)$, where i is the firm-indicator. Let q^0 and q^a be the abated amounts under absolute control (full abatement) and under standards, respectively. Assume firms are all in compliance at the beginning of the state, and the initial permit held for all firms is $q^0 - q^a$.

Future permit prices is a random variable, with probability density function $f(p)$, and $E(p) = \mu$. Under the certainty scenario, $\Pr(p = \mu) = 1$. The firms' revenue R is exogenous. A firm chooses to maximize its expected utility $u[\Pi(q_i)]$, where $\Pi = R + \int_{q^a}^{q_i}\left[p - a_i(q)\right]dq$ is the firm's total profit, coming from the exogenous revenue and the permit sales (either positive if the firm is a permit seller, or negative as a buyer):

$$E[u(\Pi(q_i))] = \int_0^\infty u\left[R + \int_{q^a}^{q_i}[p - a_i(q)]dq\right]f(p)dp$$

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[8] In the paper they addressed this as "compliance decision," yet there's no compliance issue in the model. The firms were all assumed to be in compliance (i.e., no more discharge than their permit holding).

The FOC is:[9]

$$E\left[u'(\Pi(q_i^u))[p - a_i(q_i^u)]\right] = 0$$

Assume the firms are risk averse, i.e. $u' > 0$ and $u'' < 0$. From this, they made the following proposition:

Proposition 1

Under uncertainty, a risk averse permit seller would abate less (i.e., withholding more permits to sell), while a risk averse buyer would abate more (i.e., demanding fewer permits).

Following from the proposition, the eventual permit market equilibrium price is indeterminate, depending on the magnitude of shifts of demand and supply. The authors then use experiment to test the permit market equilibrium hypotheses as stated in the beginning of this section. They found that in the lab setting environment, under uncertainty, permit price is almost the same as under certainty; the permit trading volume does not fall compared to that under certainty, and the market efficiency seemed to be the same. They argued the results might be due to the irreversibility of investment in abatement technology; when permit price fluctuates, buyers who have made all the abatement investment may still abate less (or at least the same amount) until the uncertainty is eliminated. A similar rational can be used to explain the lack of efficiency loss. Firms tend to adopt the "wait-and-see" strategy when they have invested in irreversible abatement technology; due to the permit price uncertainty, firms may also be uncertain about their roles as permit buyers or sellers in the market. These concerns

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[9] Note that the FOC becomes $\mu - a_i(q_i^c) = 0$ when there's no uncertainty.

may in turn affect firms abatement decisions, and thus lead to the experimental result that the efficiency level remains unchanged under uncertainty.

2. Risk aversion and compliance

2.1 Stranlund, 2008

In this paper Stranlund explores the role of risk averse firms' exogenous characteristics in the permit market. His model is similar to Stranlund and Dhanda (1999) who analyzed the effects of (exogenous) firm-specific factors on regulators' enforcement decision. In that paper, Stranlund and Dhanda argued that, as in previous compliance and enforcement literature, the firms' compliance decision is independent of their exogenous characteristics (e.g. the types of operation, firm sizes, etc.) This statement is straightforward for risk neutral firms, because firm-level factors do not play a role in the firms' cost-benefit analysis. In this paper, Stranlund expands the model to assess the effects of the risk attitude of firms on compliance. A firm's benefit $b(\bullet)$ is a function of its emission q, and some parameter α (b is strictly increasing in both q and α). Firms differ in the benefit function, and the difference is captured in α. Each firm maximize it expected utility as:

$$\max E(u) \equiv U(w) = (1 - \pi)u(w^0) + \pi u(w^1)$$

where π is the audit probability, and $w^0 = b(q,\alpha) - p(q - v - \ell_0)$ is the profit when no violation is detected, while $w^1 = w^0 - f(v)$ is the profit when caught. $f(\bullet)$ is the penalty

function, strictly increasing and strictly convex in violation $v = q - \ell$, ℓ is the firm's permit

holding, and ℓ_0 is its initial permit endowment; p denotes the permit price[10].

As concluded in Malik's (1990) work, Stranlund also found that a firm's emission and

subsequent compliance decision is independent of the firm's risk attitude, its exogenous

parameter α, and the enforcement policy, since the FOC of the above maximization problem

suggested a firm chooses to be noncompliant if and only if $p \leq \pi f'(0)$. Yet, risk attitude may

be important *after* the firm has decided whether to be compliant or not.

From the FOC:

$$\frac{\partial U}{\partial v} = (1 - \pi) u'(w^0) p + \pi u'(w^1) \left[p - f(v) \right] \leq 0; \quad \text{with "=" if } v > 0$$

which can be rewritten as:

$$p - \pi f'(v) R(v, \alpha, \ell_0, \pi, p) \leq 0; \quad \text{with "=" if } v > 0$$

$$R(v, \alpha, \ell_0, \pi, p) = \frac{u'(w^1)}{(1 - \pi) u'(w^0) + \pi u'(w^1)} = \frac{u'(w^1)}{U'(w)}.$$

A noncompliant firm's violation can be defined as $\bar{v} = \bar{v}(\alpha, \ell_0, \pi, p)$, and the SOSC for

an optimal violation requires $\pi f'(v) R(v, \alpha, \ell_0, \pi, p)$ to be strictly increasing in v[11].

The function $R(\cdot)$ can be seen as an adjustment to the expected marginal penalty $\pi f'(v)$,

which accounts for the firm's risk attitude. When a firm is risk neutral (linear utility function),

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[10] In the model, the only uncertainty the firms face is whether they will be audited.

[11] This condition, which can be expressed as $f''R + f'R_v > 0$, holds if the firm is not a risk seeker.

R=1; with risk aversion, it can be shown that R > 1. Stranlund argued that it can be shown that, given a firm is noncompliant in the first place, its scale of violation would then be affected by its risk attitude. Since the expected marginal penalty is higher for a risk averse firm (because R > 1), for any two otherwise identical noncompliant firms, the risk averse firm would choose a lower level of violation than the risk neutral firm.

Note that when a firm is noncompliant, $p - \pi f'(\bar{v})R(\bar{v},\alpha,\ell_0,\pi,p) = 0$; it can then be derived that the sign of $\bar{v}_\alpha$ and $\bar{v}_{\ell_0}$ is the same as the sign of $-R_\alpha$ and $-R_{\ell_0}$, respectively.

The following proposition follows from the analysis:

Proposition 2

A noncompliant firm's violation is increasing (decreasing) in its initial permit endowment and its benefit from emission parameter α if it is risk averse, and exhibits decreasing (increasing) absolute risk aversion. When the firm exhibits constant absolute risk aversion, its violation is independent of the permit endowment and the parameter α.

Stranlund discussed the firms' risk attitude on the permit market equilibrium as well. When the market has mainly noncompliant risk averse firms, he argued, compared to a noncompliant market in which most of the firms are risk neutral, the permit equilibrium price would be higher (holding total permit fixed), with lower aggregate emission and a higher degree of compliance. These results follow from the above propositions. Note that a risk averse noncompliant firm would choose a lower level of violation, through demanding more permits[12]. As demand goes up, permit price becomes higher, and firms would then reduce

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[12] A firm's emission depends only on the permit price in the model.

their emission further. As a result, the eventual equilibrium emission level decreases, and the degree of compliance and permit price both go up. Yet Stranlund also pointed out that, the increased permit price would induce higher violation level of noncompliant risk neutral firms[13].

2.2 Ozanne and White [OW], 2008

OW used contract theory to examine the impact of risk aversion on farmers' behavior when facing government regulation on input usage in the form of a quota, while the government can only detect violations with some probability $0 < p < 1$.

To look into these effects, OW constructed a model in which the regulator devises a contract that regulates the harmful input usage through a quota system, through which participating farmers get compensation payments or pay fines for quota violation, in order to maximize social welfare. The social welfare function is made up of three major terms: the environmental benefit   (cost) of using the input x, $v(x)$ (with $v' < 0, v'' \geq 0$), the expected producer surplus from participation, and the net transfer payment adjusted for the shadow price of public funds:

$$\max_{b,s,p} EU = v(x) + \left[(1-p)u(b + R(x)) + pu(b + R(x) - f(x-s)) - u(\bar{R})\right] - (1+\tau)[b + mp - pf(x-$$

where $b \geq 0$ is the transfer payment to farmers who participate in the quota program, $R(\bullet)$ the profit function in $x \geq 0$ (with $\bar{R} = R(\bar{x})$ as the maximum profit in the absence of

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[13] Keep in mind that the market is has mostly risk averse firms. The increased violation by the risk neutral firms does not offset the total decreased violation by the majority of firms.

regulation), f the unit penalty for violation, $s \geq 0$ the quota level, and m denote audit rate and monitoring cost. u is the farmer's utility function.

The social welfare maximization problem is subject to two constraints: the individual rationality (IR) constraint, namely,

$$(1-p)u\big(b+R(x)\big)+pu\big(b+R(x)-f(x-s)\big)\geq u(\bar{R}),$$

which specify the condition in which the farmer prefers to participate; and the incentive compatibility (IC) constraint, i.e., the farmer weakly prefers participation and noncompliance at level $x \geq s$ to that at level $\tilde{x} \geq x \geq s$,

$$(1-p)u\big(b+R(x)\big)+pu\big(b+R(x)-f(x-s)\big)-u(\bar{R})\geq(1-p)u\big(b+R(\tilde{x})\big)+pu\big(b+R(\tilde{x})-f(\tilde{x}-s)\big)-u(\bar{R}).$$

Using Taylor series expansion, one can show that at the limit when $\tilde{x} \to x$, the IC constraint reduces to:

$$(1-p)u'\big(b+R(x)\big)R'(x)+pu'\big(b+R(x)-f(x-s)\big)\big(R'(x)-f\big)\leq 0.$$

The government's optimal contract is (b^*, s^*, p^*) such that the FOC is satisfied:

$$\pi'(x^*)=\frac{-v'(x^*)}{1+\tau}+\frac{A\big[C+\big(m-f(x^*-s^*)\big)A\big]}{AD-BC}\pi''(x^*), \text{ where}$$

$$A=(1-p^*)u'\big(b^*+\pi(x^*)\big)+p^*u'\big(b^*+\pi(x^*)-f(x^*-s^*)\big)$$
$$B=(1-p^*)u''\big(b^*+\pi(x^*)\big)\pi'(x^*)+p^*u''\big(b^*+\pi(x^*)-f(x^*-s^*)\big)\big(\pi'(x^*)-f\big)$$
$$C=u\big(b^*+\pi(x^*)\big)-u\big(b^*+\pi(x^*)-f(x^*-s^*)\big)$$
$$D=u'\big(b^*+\pi(x^*)\big)\pi'(x^*)+u'\big(b^*+\pi(x^*)-f(x^*-s^*)\big)\big(\pi'(x^*)-f\big)$$

From the FOC, one can see that there is a trade-off between the monitoring cost m and the pollutant abatement (in the form of a quota); it is also conditioned by the farmers' risk attitude[14]. That is, compared to the risk neutral case, there is no clear relationship to be drawn about the trade-off or efficiency level of this environmental contract due to farmers' risk averse nature.

OW later asserted that, if the government's contract is such that the quota level s is exactly equal to the farmers' intended input usage level x, then the risk attitude no longer complicates things; since the contract induces perfect compliance, farmers' expected fine is zero and the welfare function, IR and IC constraints become:

$$\max_{b,s,p} EU = v(x) + \left[u\big(b + R(x)\big) - u(\bar{R}) \right] - (1+\tau)(b+mp)$$
$$IR: \ u\big(b + R(x)\big) \geq u(\bar{R}) \ \Rightarrow \ b + R(x) \geq \bar{R}$$
$$IC: \ pfu'\big(b + R(x)\big) \geq R'(x)u'\big(b + R(x)\big) \ \Rightarrow \ pf \geq R'(x)$$

And the solution is:

$$\pi'(\hat{x}) = \frac{-v'(\hat{x})}{1+\tau} + \frac{m}{f}\pi''(\hat{x}),$$

which is exactly the same as in the risk-neutral case. One can easily derive the comparative statics, defining $h = m / f$:

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[14] Note that with risk neutrality, the FOC is reduced to: $\pi'(\hat{x}) = \dfrac{-v'(\hat{x})}{1+\tau} + \dfrac{m}{f}\pi''(\hat{x})$. If monitoring is costless, then the FOC is simply $\pi'(x_0) = \dfrac{-v'(x_0)}{1+\tau}$. Then by concavity of $\pi(\bullet)$, $\hat{x} > x_0$: moral hazard due to costly monitoring reduces the efficiency of the pollution-control scheme under risk neutrality.

$$\frac{dx}{dh} = \frac{(1+\tau)\pi''(\hat{x})}{(1+\tau)\pi''(\hat{x}) + v''(\hat{x}) - (1+\tau)h\pi'''(\hat{x})}.$$

Then, if further assuming that the environmental cost function v is linear, and $\pi''' > 0$ (e.g., a strictly concave profit function from Cobb-Douglas production), then as the monitoring cost-fine ratio goes up, the optimal input usage and the quota increase[15].

3. Monitoring and Fines

While the uncertainty and risk attitude play a role in determining the efficiency of the permit markets, the structure of the penalty the regulator uses to deter noncompliance cannot be ignored. In some early literature on crime economics, it is argued that the optimal solution for crime deterrence is to set the fine as high as possible, e.g. setting it equal to the individual's wealth, and minimize the monitoring costs (since monitoring is costly.) Further research on this matter, however, suggests that this argument fails to consider many other significant factors influencing the regulations on illegal behavior; in particular, the solution in which the fine expands to its upper limit may not be efficient. Here I review some papers that have explored these issues, in order to provide some insights for the boundary and structure on penalties, and government's regulations.

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[15] Although result is more intuitive in the case where $s = x$, I find the assumption is not very practical for environmental quality improvement. The goal of pollution control is to rebate the input usage, and setting the quota equal to the farmers' expected usage seems not so compatible with the policy's purpose.

3.1 Polinsky and Shavell [PS], 1979

As PS first pointed out in their model, when individuals are risk averse (in contrast to risk neutral), setting the fine to its upper bound may not be optimal. Instead, if the auditing cost is low enough, then at optimum, the audit probability should approach 1 and the fine should be set to the gains from committing illegal activities[16]. Formally, they presume the individuals may be either type-*a* or type-*b*, whose gains from engaging in the illegal activities are g_a and g_b, respectively, where $g_a < g_b$. The illegal activities impose external cost E on society; specifically, $g_a < E < g_b$. Individuals can insure fully against the risk of bearing the external cost, by paying a per capita premium $\sigma = nE$, where n is the proportion of law-breaking population. The government finances its efforts to catch these illegal activities (with success rate p) through per capita tax, t and the fines collected. Per capita tax is defined as: $t = c(p,\lambda) - npf$, where f denotes the fine amount, and $c(p,\lambda)$ is the per capita cost, as λ is a shifting parameter such that $c_\lambda > 0$ and $c(p,0) = 0$. The expected social welfare is:

$$EU = qEU_a + (1-q)EU_b,$$

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[16] They also concurred that, when individuals are risk neutral, the optimal policy is to set the audit probability as low as possible, and the fine as high as possible, regardless of the cost to audit.

where q is the proportion of type-a individuals, with $0 < q < 1$; EU_a and EU_b are expected utility of type-a and -b individuals, respectively[17]. Each individual has initial wealth of y, and chooses whether to violate the law if and only if:

$$(1-p)U(y-t-\sigma+g_i)+pU(y-t-\sigma+g_i-f)>U(y-t), \quad i=a,b.$$

The "threshold probability," below which it is impossible to deter violations, can then be derived using the following equation (assuming the maximum fine is the individual's wealth w, and the gain of committing the crime is g) [18]:

$$(1-\bar{p})U(w+g_i)+\bar{p}U(g_i)=U(w), \quad i=a,b \quad \Rightarrow \quad \bar{p}(w,g_i)=\frac{U(w+g_i)-U(w)}{U(w+g_i)-U(g_i)}, \text{ and}$$

$$\bar{p}_g=\frac{U'(w+g_i)[U(w)-U(g_i)]+U'(g_i)[U(w+g_i)-U(w)]}{\left[U(w+g_i)-U(g_i)\right]^2}>0$$

That is, as the private gain increases, the minimal deterring probability gets higher (consistent with intuition.) The following proposition affirms Becker's (1968) statement:

Proposition 3.1

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[17] In the original setup, PS defined this as "ex-ante" expected utility for an individual, where they assumed the individual did not know his/her type prior to violating the law (so q, as defined in the original model, was the probability of an individual being type-a). Yet in later discussion of the paper, PS derived their results as if there were 2 types of individuals who knew their types when deciding to commit the crime or not, and this function was the social expected utility function to be maximized. Here I presented the model in the form of the latter setup to avoid confusion.

[18] The implicit assumption is that $0 < g_i < w$, so that $0 < \bar{p}(w,g_i) < 1$.

(1) If individuals are risk neutral, and suppose it is optimal to control the law-breaking activities (i.e., $p > 0$), then the optimal audit probability, p^*, is equal to $\bar{p}(y - t - \sigma, g_a)$, combining with an optimal fine f^* equal to the law-breaking individual's wealth.

(2) In equilibrium, only type-b individuals engage in illegal activities.

Note that this result holds even when the enforcement cost is sufficiently low. $\bar{p}(w, g_a)$ is the threshold probability of type-a individual, whose gain from the illegal act is the least in the society. When the optimal $p^* > 0$, it must be the case that only type-b individuals engage in illegal activities[19]. If $p^* > \bar{p}(y - t - \sigma, g_a)$, it can then be shown it contradicts the fact that (p^*, f^*) is the optimal policy.

PS then discuss the case when individuals are risk averse and make the following proposition.

Proposition 3.2

(1) Suppose individuals are risk averse. When λ is sufficiently low, the optimal audit rate $p^* = 1$, and optimal fine f^* is equal to g_b.

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[19] If all individuals engage in the illegal activities, then it is clear that it is optimal to set $p^* = 0$ (since the regulators cannot deter any crime). On the other hand, whenever type-a individuals are not deterred, it must be the case that type-b individuals are not deterred either, since $g_b > g_a$. If everyone is deterred at the optimal (p^*, f^*), there must exist some fine $f^0 < f^*$ which can induce the type-b individuals to engage in the activities, and the social (expected) welfare is higher under (p^*, f^0):

$$EU(p^*, f^*) = y - c(p^*, \lambda) < EU(p^*, f^0) = y - c(p^*, \lambda) + (1 - q)(g_b - E),$$

for at (p^*, f^0), the risk premium $\pi = (1 - q)E$, and tax $t = c(p^*, \lambda) - (1 - q)p^* f^0$. Note that when individuals are risk neutral, the utility function is linear in wealth; the above derivation follows.

(2) When the gains from controlling the violations are sufficiently small (i.e.,
$\left(g_b - g_a\right) \to 0$), $p^* \to 1$ *and* $f^* \to g_b$.

When $\lambda = 0$, the regulator can monitor/audit the violations at zero cost, so the audit probability should equal 1, and the optimal fine equals g_b , in order to transfer income from those who gain from the violations to those who do not violate.[20] By continuity, it can then be shown that as $\lambda \to 0$, $p^* \to 1$ and $f^* \to g_b$. The same rationale applies for proving the second part of this proposition.

As PS indicated, when there are other factors in play such as risk aversion, the probability of catching a law-violating act is not necessarily equal to zero; in fact, as shown in the model, when risk averse individuals can insure against the externality, and the enforcement cost is low enough, the audit probability will approach 1, while the penalty approaches the gain from violations. There are other factors that might be in play as well. For instance, PS assumed in the model that the regulators can perfectly and effortlessly observe individuals' types and act accordingly.[21]

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[20] Higher penalty implies more risk to bear, and the disutility from the risk would thus be larger than the saving on the enforcement cost. Though a fine higher than E is sufficient to just induce violation of type-b individuals, setting it equal to g_b allows just income transfer.

[21] In a later paper by Malik (1990), he developed the idea of imperfect screening where the regulators had to invest to find out the individuals' types under the assumption of risk neutrality. This paper is not in the scope of this review.

3.2 Bontems and Rotillon [BR], 2007

In PS' paper, there is no faulty conviction on the regulator's side, i.e., the probability of being wrongly fined is zero. To assess this aspect of the enforcement issue, the paper by BR offers an insightful perspective. Their model is to assess risk averse individuals' compliance decision to environmental standards, where the individuals are subject to some social norms (or sanctions) that deter them from being noncompliant. Those heterogeneous individuals differ in private compliance costs, and their degree of risk aversion toward noncompliance; the latter is influenced by social norms in the sense that, when the (expected) rate of noncompliance is high, ceteris paribus, an individual's cost of being caught for noncompliance is low.

To see the effects of those factors on market outcome, BR's model specifies a population of heterogeneous individuals who, facing the given environmental standards, can either spend a compliance cost, *c*, or nothing. The regulators audit individuals' behavior with probability *p*. There is also a probability μ of being wrongly convicted of noncompliance.[22] Let *f* and *F* with $f \leq F$ be the fines for involuntary noncompliance and voluntary violation, respectively.[23]

The revenue from violation is $R = r - \delta$ where $r \geq 0$ is the potential maximum revenue, and

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[22] The probability of involuntary convicted noncompliance is μp .

[23] This implicitly assumes the government can distinguish involuntary violation from voluntary noncompliance effortlessly. But there is no constraint on the equality of f and F.

$\delta \leq r$ is the loss (e.g., market sanction) from noncompliance[24]. Let the market's expected

compliance rate be τ; the revenue from an individual that is not convicted is:[25]

$$V = (r - \delta)\Pr(noncompliant \mid not\ convicted) + r\Pr(compliant \mid not\ convicted)$$
$$= \frac{(1 - \tau)(1 - p) + \mu\tau(1 - p)}{(1 - p)(1 - \tau) + \tau(1 - \mu) + \tau\mu(1 - p)}(r - \delta) + \frac{\tau(1 - \mu)}{(1 - p)(1 - \tau) + \tau(1 - \mu) + \tau\mu(1 - p)}r$$
$$= r - \frac{(1 - \tau + \mu\tau)(1 - p)}{1 - p + \tau p(1 - \mu)}\delta \quad \in (r - \delta, r)$$

Noncompliant individuals also suffer a "psychic cost" from social norm. Let θ be the

type (i.e., degree of adherence to social norm) of an individual, and the "social sanction" is

defined as $\theta\psi(\tau)$; ψ is increasing in the compliance rate, and $\theta \geq 0$ [26].

An individual of type (θ, c) chooses to be compliant if and only if the expected utility

from compliance is greater than that from noncompliance:

$$(1 - \mu p)U\big(V(\tau) - c\big) + \mu p U\big(R - f - c\big) > p U\big(R - F\big) + (1 - p)U\big(V(\tau)\big) - \theta\psi(\tau)$$
$$\Rightarrow \theta > \tilde{\theta}(c, \tau) \equiv \frac{\Delta W(c, \tau)}{\psi(\tau)},$$

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[24] It is a bit awkward to assume that violation is subject to a loss. But one can imagine the situation in which such a violation creates an externality that everyone suffers, or in which the violation itself causes the individual's reputation to drop, and thus suffers a loss.

[25] V is increasing in τ and p, but decreasing in μ.

[26] Thus an individual with higher θ is less willing to be noncompliant.

where $\triangle W(c,\tau) = pU(R-F)+(1-p)U\big(V(\tau)\big)-(1-\mu p)U\big(V(\tau)-c\big)-\mu pU\big(R-f-c\big)$ is the

expected utility gain from noncompliance, and $\tilde{\theta}(c,\tau)$ is the minimal adherence for an

individual to choose compliance.[27]

Through rational expectation, if everyone in the market anticipates a compliance rate τ,

τ must be equal to the share of population $H(\tau)$ which is compliant. The market equilibrium

compliance rate can then be solved as:

$$\tau^* = H(\tau) \equiv \int_{\underline{c}}^{\bar{c}} \int_{\tilde{\theta}(c,\tau)}^{\bar{\theta}} dG(\theta,c),$$

where $G(\bullet)$ denotes the joint distribution of (θ,c) over the product of their support. It can

then be shown that there exists at least one solution that satisfies the above equation[28].

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[27] It can be shown that $\triangle W(c,\tau)$ is increasing in f (fine for involuntary violation) and μ, decreasing in F (fine for voluntary noncompliance), and is ambiguous with the loss from violation δ and audit rate p (given that $\mu > 0$, and individuals are risk averse.)

[28] First BR showed that $\tilde{\theta}(c,\tau)$ is continuous and increasing in c and decreasing in τ; it follows that H is continuous and non-decreasing in τ. The existence of an equilibrium then follows by intermediate theorem. They did not, however, provided specific boundaries for c and θ. However, they did not specify the bounds for the compliance cost c or the type variable θ.

Proposition 3.3

An increase in audit rate p or the market loss from violation, δ , could actually induce a decrease in the equilibrium compliance rate, while the equilibrium compliance rate increases as the enforcement error (i.e. the probability of being wrongfully convicted of violation, μ) gets higher.

The above statement follows directly from the properties of $_\triangle W(c,\tau)$ (see footnote 27).

The enforcement error also influence the truth-telling process. For simplicity in the discussion of information disclosure, BR assessed the case when the social norms are not binding[29], i.e., $\theta = 0$; in that case, there exists a unique $\tilde{c}(\tau)$ such that individuals with $c \le \tilde{c}(\tau)$ choose to comply with regulations, and the equilibrium compliance rate is reduced to:

$$\hat{\tau} = \int_{\underline{c}}^{\tilde{c}(\tau)} dG(0,c).$$

Again, risk aversion implies the loss from violation, δ , which has an ambiguous impact on the equilibrium compliance rate.

Consider a policy requiring all individuals to report their law-abiding status. An individual who is not compliant is fined the amount of $s \le f$ if he truthfully reports noncompliance, and $S \le F$ if he reports being in compliance. An involuntarily noncompliant individual will disclose the true status if and only if:

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[29] When there are no social norm constraints, compliance exists only if $f < F$.

$$U(R-s-c) > pU(R-f-c) + (1-p)U(V(\tau)-c)$$

Proposition 3.4

A "rebate" for truth-telling is necessary for involuntary noncompliant agents to disclose their true status; that is, $s < f$.

Since for $f = s$, the decision rule for truth-telling becomes:

$U(R-f-c) > U(V(\tau)-c)$, and for monotonic $U(\cdot)$, $R < V(\tau)$ implies no one reports violation. The statement then follows.

3.3 Arguedas, 2008

While there are many papers on compliance issues of environment control, there are only a few that discuss both the compliance and the regulation mechanism. Arguedas used a model that incorporated two factors of sanction. He defined one of them as the "gravity-based" component, which is directly related to the violation level, and the involved liability and mitigation efforts. The other one is the "non-gravity-based" part, which concerns the penalty's own economic impact on violators, or other legal/justice matters. The major difference in this paper is that the environmental policy in the model is endogenously determined, and thus results in the unusual conclusion that the optimal standard/policy may either induce compliance or noncompliance, depending on the monitoring costs and sanction scale. When

the firms face sufficiently large gravity-based sanction component, Arguedas argued, the optimal policy will induce noncompliance instead[30].

In the model, firms' benefit *b* is a function of their pollution *e*, with b(0) = 0 and $b'''(e) \geq 0$ but sufficiently small; *d(e)* denotes the pollution's damage function, which is strictly increasing and convex in *e*, with *d(0) = 0*. The regulator imposes an environmental standard *s*, and enforces it by inspecting firms' compliance. The cost of inspection is *c > 0*, with *p* as the probability of auditing. The penalty function is defined as:

$$f(e-s) = \begin{cases} a + g(e-s) + h(e-s)^2 & \text{if } e-s > 0 \\ 0 & \text{if } e-s \leq 0 \end{cases}, \quad a \geq 0, g > 0, h \geq 0 \text{ and } g^2 > 2ah,$$

Note that *a* is the non-gravity-based component in the penalty function, and if *a > 0*, then the penalty is discontinuous at 0. The gravity-based component is captured by *g* and *h*.

To find the optimal policy, Arguedas first solves for the firm's best response to a given set of standards and audit probability, *{s, p}*, and then uses that to determine the policy that maximize social welfare.

Let $e(s,p)$ the firm's optimal pollution level that solves the firm's profit maximizing problem:

$$\max_{e \geq 0} b(e) - pf(e-s).$$

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[30] As the gravity-based penalty increases, the violation to certain standard decreases. Then by convexity of the penalty function, as the standard goes down, the decrease in the noncompliance with respect to the increased gravity-based sanction grows. Consequently, it can be shown that the optimal policy (the least-cost policy) in the model is the one that induce noncompliance.

And let $\tilde{e}$ be the pollution level in the absence of regulation. Note that $e(s,p) \leq \tilde{e}$.

Taken the firm's response as given, the regulator solves for the social welfare maximizing problem:

$$SW(s,p) = P(s,p) - d(e(s,p)) + pf(e(s,p)-s) - pc, \quad P(s,p) \equiv \max_{e \geq 0}\{b(e) - pf(e-s)\}$$
$$\Rightarrow \max_{s,p} SW = b(e) - d(e) - pc \quad s.t. \ e = e(s,p)$$

In this problem, the penalty is simply a lump-sum transfer and thus does not distort the players' behavior, and thus does not enter the optimization problem; there is no budget constraint on the regulator's side either.

The regulator has two instrument to induce certain amount of pollution, namely the standard s and the audit probability p.[31] A compliance policy is that the regulator announces a standard $s = e$, and the corresponding auditing probability to induce the compliance, denoted as $p^c(e)$; the noncompliance policy is when the regulator sets the standard to 0, and chooses a corresponding auditing probability $p^n(e)$ such that the firm still pollutes to the desired pollution level e. Since the pollution level chosen by the firm is increasing in the standard[32] and decreasing in the audit probability, given a set of policy $\{s,p\}$ in which $s > 0$ that induces pollution level e, there exists another set of policy $\{s',p'\}$ such that

_____

[31] Arguedas proved that each possible level of pollution $e \leq \tilde{e}$ can be induced by either a compliance or a noncompliance policy.

[32] This is true only when the penalty's gravity-based component is convex. In the case of linear gravity-based component, the optimal noncompliance decision does not vary with standard.

$s' < s$ and $p' < p$, yet that induces the same amount of pollution e. The following proposition concludes this argument:

Proposition 3.5

(i) If the optimal policy $\{s^*, p^*\}$ induces compliance, then $s^* > e^w$, where e^w is the efficient pollution level in the absence of enforcement cost c, and $p^* < p^c(e^w)$

(ii) If the optimal policy $\{s^*, p^*\}$ induces noncompliance, there are 2 possible scenarios:

(a) if the gravity-based component is strictly convex in the degree of violation, then $s^* = 0$, $p^* < p^n(e^w)$ and $e(0, p^*) > e^w$

(b) If the gravity-based component is linear in violation, then $s^* \in [0, \bar{s})$, where $\bar{s}$ is a standard such that $p^* = p^c(\bar{s}) < p^n(e^w)$, and $e(0, p^*) > e^w$.

Thus, as monitoring is costly, the regulator's policy is a trade-off between efficiency and the monitoring costs, and at the optimum, it must be the case that the marginal loss in efficiency is equal to the marginal saving in enforcement/monitoring cost.

Furthermore, Arguedas then proved that in the case of a linear gravity-based component, the optimal policy always induces compliance (or, in other words, the optimal policy is always the compliance policy); while in the case of a strictly convex gravity-based component, the optimal policy depends on the non-gravity-based part of the penalty function.[33]

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[33] When the non-gravity-based part is small enough such that for all $e < \tilde{e}$, $p^c(e) > p^n(e)$, then it is optimal to have the noncompliance policy; when it is large enough such that for all $e < \tilde{e}$, $p^c(e) < p^n(e)$, then the optimal policy must induce compliance. When the non-gravity-based component is in between the 2 cases, if the monitoring cost is sufficiently large, or the gravity-based penalty is sufficiently high, then it is optimal to have a noncompliance policy.

These results are obtained based on the key assumption that the optimal policy/standard is endogenously determined, and the penalty function has the possible discontinuity at zero violation due to the non-gravity-based part. Though the idea is innovative, it is a bit moot to assume there is a penalty in the absence of violation. It is also uncertain that the results would hold under a permit scheme.

4. Pollution Tax and Industry Size[34]

Though we are more interested in the tradable permit markets for pollution, emission tax is also widely adopted and somewhat equivalent to tradable permits in some rather stringent cases. The literature on pollution tax can, indeed, shed some light on how environmental regulations could facilitate the industry's growth or a firm's doom.

4.1 Katoulacos and Xepapadeas [KX], 1995

The paper by KX explored the role of pollution tax under an oligopolistic market structure. By examining both the fixed number of firms and the free-entry setup, they identified the regulatory efforts the government can take on to increase social welfare. When the number of firms is fixed, and because oligopolistic output is suboptimal from that of perfect competition, they confirmed the second-best pollution tax should be less than the marginal social damage, though it will increase as the industry size grows. When the number of firms is endogenous, however, the second-best emission tax is even greater than the

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[34] This section is added after the first oral to reflect committee members' suggestions.

marginal social damage, for large tax could bar potential entrants (and thus restrict the degree of pollution), such that the industry size is closer to that at social optimum.

Assuming there are N firms in the industry, each is producing some homogenous product of quantity q and some kind of pollution e. An individual firm's cost function is $C(q,w)$, where w is the abatement effort a firm adopts. Suppose the government imposes some pollution tax τ, then a firm's profit function is: $\Pi = pq_i - C(q_i, w_i) - F - \tau e_i$, where F is the fixed cost. To simplify the analysis, KX assumed a linear inverse product demand function: $p = P(Q) = a - Q$, where Q is the aggregate output, as well as linear (and additive) cost function for each firm: $C(q_i, w_i) = cq_i + gw_i$. Social damage depends on the aggregate pollution, $S(E) = N \cdot e$, assuming $e_i = vq_i + \beta w_i^{-\gamma}$ is the individual emission.[35] Then the Nash equilibrium output is:

$$q^* = \frac{a - c - v\tau}{1 + N},$$

and the optimal abatement effort is:

$$w^* = k\tau^\delta, \text{ where } k = \left(\frac{\beta\gamma}{g}\right)^\delta \text{ and } \delta = \frac{1}{1+\gamma}.$$

Social welfare is the sum of the consumer's and the producer's surplus:

$$V(N, \tau) = \int_0^{Nq} P(x)dx - N \cdot c(q, w) - S(E) - N \cdot F.$$

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[35] The parameters: $a, \beta, c, g, v, \gamma$ are all assumed to be positive.

When the number of firms is determined endogenously, the optimal pollution tax is simply:

$$\tau^* = \arg\max_{\tau} V(N(\tau), \tau),$$

where both the free entry condition: $\Pi = 0$ and the FOC for τ : $\dfrac{\partial V}{\partial \tau} + V_N\left(\dfrac{\partial N}{\partial \tau}\right) = 0$ must

be satisfied. KX then postulate the following:[36]

Proposition 4.1

When the market structure is endogenous, the optimal pollution tax may exceed marginal social damage.

Note that when the industry size is endogenous, a positive pollution tax not only suppresses pollution (and production), but helps bringing the number of firms closer to that of the social optimum. If the benefit of the pollution tax is strong, then it could be the case that the optimal pollution tax exceeds marginal social damage.

KX also claims that, if the government impose some kind of lump-sum "license fee" in addition to an under-internalized emission tax, which helps to bring down the number of firms closer to the second-best optimum, it can increase the social welfare more, comparing to simply imposing a single emission tax which exceeding the marginal damage.

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[36] The N that satisfies $V_N = V_N\left(N(\tau), \tau\right) = 0$ is defined as the "second-best", or "constrained" optimal, which is typically less than the free-entry Nash equilibrium when there are no regulations.

4.2 Requate, 1997

Like KX, Requate also studied the impacts of pollution tax on firms' behavior when the number of firms is endogenous. He modeled the oligopolistic markets where firms engaging in Cournot competition and with partial equilibrium, and in contrast to what KX (1995) found, he claimed that with the free entry condition, the scale of optimal emission tax with respect to marginal pollution damage depends heavily on the curvature of the demand function, as well as the complementarity between emission and production. In the case where oligopolistic firms are identical and the industry size exogenous, it has been established that the second-best Pigouvian pollution tax should be smaller than the marginal social damage to induce production. If the number of firms are endogenous, nonetheless, there is no immediate relation between the tax and marginal social welfare if the tax is the only instrument the government uses for both the oligopoly and the pollution. To illustrate the situation, assuming there are n identical firms in the market, the cost function of an individual firm is:

$$C(q) = \begin{cases} v(q) + F & \forall\, q > 0 \\ 0 & \forall\, q = 0 \end{cases}$$

when there is no abatement technology, and

$$C(q, e) = \begin{cases} v(q, e) + F & \forall\, q > 0 \\ 0 & \forall\, q = 0 \end{cases}$$

when abatement is possible, where e is the firm's emission. When there is no abatement, a firm's pollution is proportional to its output q, i.e. $e = d \cdot q$, with $d > 0$. In the case where abatement is possible, the production and the emission are both a firm's decision variables. Particularly, its marginal variable cost is decreasing emission, while the marginal benefit of

emission is decreasing, i.e., the cost function is convex, which satisfies:

$$v_q > 0, v_{qq} \geq 0, v_{ee} \geq 0, v_{qe} \leq 0$$

and

$$v_{qq}v_{ee} - v_{qe}^2 \geq 0.$$

Requate uses a partial equilibrium model to illustrate the change of the industry size, with identical firms. Let the Pigouvian tax rate be τ, and the product demand function be $P(Q)$[37], where Q is the aggregate production, $Q = n \cdot q$, the Cournot-Nash equilibrium when abatement is not possible, is then defined by the following conditions:

$$P'(Q^*)q^* + P(Q^*) - v'(q^*) - \tau d = 0,$$

and the free-entry is described as:

$$P(n^*q^*)q^* - v(q^*) - F - \tau d \cdot q^* = 0.$$

In the other case when abatement is available, the Nash equilibrium is defined by:

$$P'(Q^*)q^* + P(Q^*) - v_q(q^*, e^*) = 0$$

$$-v_e(q^*, e^*) = \tau$$

and the free entry condition is:

$$P(n^*q^*)q^* - v(q^*, e^*) - F - \tau e^* = 0.$$

To find the optimal tax rate in the oligopolistic market, the government maximize the

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[37] There exists a choke-off price $\bar{p}$. $P' < 0$ and $P(\cdot)$ is not too convex which satisfies $\dfrac{P''(Q)}{P'(Q)} \cdot Q > -1$.

economy's welfare:

$$W(\tau) = \int_0^{Q(\tau)} P(z)dz - n(\tau) \cdot \{v(q^*, e^*) + F\} - S(E(\tau)),$$

where $Q(\tau) = n(\tau)q(\tau)$ the aggregate production, $E(\tau) = n(\tau)e(\tau)$ the aggregate pollution and $S(\cdot)$ the social damage function from pollution.

From the FOC w.r.t. τ, the (second best) optimal pollution tax is:

$$\tau^* = S'(E) + \frac{P\prime(Q)Q \cdot q\prime}{E\prime},$$

where in the case of endogenous number of firms and abatement technology, $E' = n'e + ne'$, while in the case which abatement is absent, $E' = (dnq)' = d(n'q + nq')$.

The relation between the second-best tax rate and marginal social damage, however, depends largely on the curvature of the demand function, $P(Q)$, and the complementarity between production q and the emission e, which in turn, depends on the assumption of availability of abatement technology.

Differentiating the FOC for firms, and with some manipulations, Requate draws the following conclusions.

Proposition 4.2 (the case when abatement technology is available)

If the demand function P is non-convex, i.e. $P'' \leq 0$, and the term: $v_{ee}e + v_{qe}q > 0$:

1) As the tax rate rises, a firm's production increases, while the aggregate production

and pollution fall. The equilibrium number of firms can then be inferred to be decreasing in the pollution tax.[38]

2) *The optimal pollution tax rate is greater than the marginal pollution damage, i.e.*

$\tau^* > S'(E)$.

Proposition 4.3 *(the case when abatement is not present)*

1) *Aggregate production and pollution both fall in response to an increase of pollution tax, regardless of the demand function's curvature.*[39]

2) *As the pollution tax rate rises, an individual firm's production is:*

   (i)   *increasing if the demand function is strictly concave, and thus the equilibrium number of firms declines unambiguously.*

   (ii)   *decreasing if the demand function is strictly convex, and the changes to the equilibrium number of firms is undetermined for* $n > 2$.

   (iii) *unchanged if the demand function is linear, and the equilibrium industry size shrinks monotonically.*[40]

3) *The second-best pollution tax is:*

   (i)   *greater than the marginal social damage when the demand is strictly concave;*

   (ii)   *smaller than the marginal social damage when the demand is strictly convex;*

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[38] In contrast to the case in which the number of firms is fixed, the increased tax drives out firms when there is free entry/exit, such that the aggregate production still falls as individual firms boost their production.

[39] Note that emission in the no-abatement case is simply a proportion to the production; and the assumption of Requate's model is such that the demand function is not too convex. See Footnote 36.

[40] When the demand function is linear, the individual firm's production remains unchanged, yet since the aggregate production declines, it must be the case that the equilibrium industry size shrinks.

(iii)   the same as the marginal social damage when the demand is linear.[41]

That is, the second-best pollution tax $\tau^* \overset{>}{\underset{<}{=}} S'(E) \,\forall\, P'' \overset{<}{\underset{>}{=}} 0$.

The above propositions offers a generalized perspective toward the pollution tax and the industry size. Note that, because the regulators use the pollution tax as a means of mitigating excess entry along with the environmental damage in the model, the second-best tax rate would typically be higher than that in a market with fixed number of firms. Moreover, whether the optimal tax in this context is greater or smaller than marginal social damage depends largely on the product demand function's curvature, as well as the complementarity between the production and emission.

In the paper Requate also discussed the scenario in which there are different types of firm. For instance, let there be two different types, each with a number of firms. Requate claims that the number of each type of firms is not unique at equilibrium, and since there are only 2 types, once the tax rate deviates from the equilibrium one, it will surely drive out one of the types of firms under free entry condition. The implication for the government, Requate claims, is that it can charge discriminating tax such that only the desired type of firms stays in the market.

4.3 Lahiri and Ono [LO], 2007

LO examine the welfare effect under pollution tax regime and the emission standard

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[41] In contrast to what KX (1995) found (Proposition 4.1): when the market structure is endogenous, if the demand is linear ($P'' = 0$), and the marginal abatement cost is independent of the level of output ($v_{qe} = 0$), the second-best tax rate exceeds the marginal damage function. Note that KX assume an additive and separable variable cost function, i.e., $v'' = 0$ in their model.

regime, for both fixed and flexible industry size. Similar to Requate's setup, they assume the market is populated with symmetric oligopolistic firms who face an inverse demand function $P(Q)$, Q the aggregate production, satisfying the assumption in footnote-36. Firms' pollution (before abatement) is a function of the production, $\theta(q)$, where the marginal pollution is greater or equal to the average pollution, i.e. $\theta'(q) \geq \frac{\theta(q)}{q}$.

Firms may abate their emission by the amount of a at abatement cost $\gamma(a)$, to meet the environmental regulations, where $\gamma(0) = 0$, $\gamma'(0) = 0$ and $\gamma'(a), \gamma''(a) > 0 \; \forall \, a > 0$. Their production cost function is increasing in the production q with increasing marginal cost. Total cost is the sum of production cost and the abatement cost[42]. The social welfare is given by:

$$W = N\pi + CS - S(E) + t \cdot E,$$

where N is the number of firms in the industry, CS stands for consumer surplus, and $S(E)$ is the social damage, a function of aggregate pollution E.[43] The last term is pollution tax transfer, and is zero under the emission standard regime.

First they show the regular results: when the number of firms is fixed, raising the unit pollution standard z (i.e., loosened environmental regulations) causes total pollution and

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[42] When facing the pollution tax, firms' total cost also include the pollution tax paid.

[43] With emission standard regime, the government imposes the unit emission limit z, and thus $E = Nzq$, while the abatement cost is $\gamma(\theta(q) - zq)$. Under pollution tax regime, a firm pollutes the amount of e pays $t \cdot e$, the aggregate emission $E = N \cdot e$, and the abatement cost is $\gamma(\theta(q) - e)$.

production to rise, as abatement cost falls. Yet as rivals' marginal costs also decrease, it is not clear whether individual firm's profit would go up or down[44]. Similar results apply to the pollution tax case: as the tax rises, total production and pollution falls, while the impact on an individual firm's profit is undetermined. Note that in both cases, production is further reduced due to either increased tax or tightened regulations; while this decrease harms the economy, it also helps to alleviate pollution damage. The eventual welfare effect depends on the magnitude of these two forces[45]. By comparing the two instruments, Lahiri and Ono conclude the following:

Proposition 4.4

When the number of firms in a Cournot oligopolistic market is fixed, tightening the emission standard will increase social welfare more than imposing an emission-equivalent tax.

Inferred from the above proposition, it can be shown that tightening the emission standard will suppress pollution more than imposing a welfare-equivalent emission tax when number of firms is fixed in the oligopolistic market.

When there is free entry, however, firms earn zero profit, and the condition also determines the size of the industry. The welfare function then becomes:

$$W = CS - S(E) + t \cdot E,$$

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[44] If the abatement cost is almost linear, and combining with the "not too convex" assumption about the demand function, they conclude single firm's profit will increase as the environmental standard loosens.

[45] They assert that, in the pollution tax case, the welfare surely decreases as aggregate pollution increases if the marginal pollution damage is greater than the pollution tax.

in which the term of aggregate profit vanishes.

Similar to Requate's assertions, Lahiri and Ono also find that an individual firm's production in the case of free entry, depends on the curvature of the demand function. In particular, when the demand function is convex, relaxing the standard will result in an increase in individual production. When the demand function is concave, individual production may be increasing or decreasing (or not changing) in accordance with relaxed emission standard. The aggregate production, though, always increases in standard relaxation. For the pollution tax regime, on some stringent condition that the average emission is constant, i.e. $\frac{\theta(q)}{q} = k$, $k \in R^+$ some constant, the individual production is increasing in pollution tax t if the product demand function is   concave, while it is decreasing in tax if product demand is convex, and unchanged if demand function is linear. That is, $\frac{dq}{dt} \overset{>}{\underset{<}{=}} 0 \; \forall \; P'' \overset{<}{\underset{>}{=}} 0$. As in the case of emission standard, the aggregate pollution is decreasing in the pollution tax[46].

Then, comparing the welfare effects of the two different schemes, they conclude the following:

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[46] This also implies that, when the product demand is non-convex (i.e. $P'' \leq 0$), then the number of firms must decline as the pollution tax increases. This result is also consistent with Requate's findings.

Proposition 4.5

1) *When the demand function is non-convex and the marginal pollution is increasing (in production), i.e. $P'' \leq 0$ and $\theta'' > 0$, a rise in pollution tax will raise welfare more than a emission-equivalent standard.*

2) *If the unit pollution is constant, i.e. $\frac{\theta(q)}{q} = k \in R^+$, then a rise in pollution tax raises the welfare:*

(i) *more than an emission-equivalent standard if the product demand is concave;*

(ii) *less than an emission-equivalent standard if the product demand is convex;*

(iii) *at the same scale as an emission-equivalent standard if the product demand is linear.*

From *Proposition 4.5*, a rise in pollution tax is welfare-superior to an emission-equivalent standard when the product demand is concave (with constant unit pollution). This is in direct contrast to *Proposition 4.4*, which stated that under fixed industry size, a tightening of emission standard is welfare-superior to an emission-equivalent tax. Similar to the case of fixed industry size, it can also be inferred from ***Proposition 4.5*** that:

(i) A rise in pollution tax can suppress more pollution than a welfare-equivalent decrease (tightening) of emission standard when the product demand is non-convex, and the marginal pollution is greater than the unit pollution, i.e., $P'' \leq 0$ and $\theta' > \frac{\theta(q)}{q}$.

(ii) A rise in pollution tax will lead to more, less or the same amount of pollution

reduction than a welfare-equivalent decrease in emission standard if the product

demand is convex, linear or concave, respectively, with constant average pollution,

$\frac{\theta(q)}{q} = k \in R^+.$

In sum, Lahiri and Ono offers a ranking to two different schemes of pollution control.

Starting from the equilibrium at which there is no environmental regulations, they find that

when the industry size is endogenous, the curvature of the product demand function is

crucial to welfare comparison of the two policy instruments.

4.4 Sengupta, 2010

While the literature reviewed in previous sections explores the industry aspect of the

emission regulations with a flexible market structure, it all assume firms are perfectly

compliant, and have not assessed the possibility of violation and its effects on welfare (and

the industry). In addition, they all assessed the industry size in a static setting. Sengupta

instead looks at the industry evolution in a dynamic way, and at each period individual firms

choose to stay or exit the market, as well as the amount of capital they are willing to invest.

Sengupta's model, drawn largely from Petrakis and Roy's (1999), specifies a set of (ex ante)

identical and competitive firms which may choose to enter the market at the beginning of

each period (with a finite total number of periods, T). Firms in the industry produce some

homogenous good at production cost $c(q_t(i))$ for firm-i in any period t, facing the inverse

product demand $P(Q)$, where Q is the aggregate output in the industry. At any period t, firms can also invest in capital which will reduce their compliance cost x_i with respect to the regulations in the market. The stock of this capital is $y_i = \sum_{t=\underline{\tau}}^{t} x_\tau(i)$, for firm-$i$ who enters the market at time $\underline{\tau}$, with $t > \underline{\tau}$. The investment cost is $\gamma(x_t(i))$, which is continuously differentiable, strictly increasing and convex with $\gamma(0) = 0$. There is no capital depreciation.

Thus, though firms in the industry are ex ante identical, the difference in the capital investment after entering creates some heterogeneity among firms, by which shake-out (exiting) may occur.

Firms incur some compliance cost with respect to the market regulations, $\varphi(q, y, \alpha)$, which depends on the production scale q, its capital stock y and the regulation parameter α, which can be interpreted as the unit pollution tax, the exogenously endowed permit amount, or some polluting standard that a firm must meet[47]. Then for a firm in the industry at any period, its total cost is simply $c(q) + \varphi(q, y, \alpha)$. The assumptions about $\varphi(\cdot)$ are:

(i) $\varphi(q, y, 0) = 0$ and $\varphi(0, y, \alpha) = 0$;

(ii) $\varphi_q, \varphi_{qq} > 0; \varphi_y \leq 0; \varphi_{yy} \geq 0$ and $\varphi_\alpha > 0; \varphi_{q\alpha} > 0; \varphi_{y\alpha}, \varphi_{qy} \leq 0$;

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[47] For instance, let $e(q, y)$ be the net emission for a firm whose production and stock of capital are q and y respectively. If α is the unit pollution tax, then $\varphi(q, y, \alpha) = \alpha \cdot e(q, y)$. If α is the permit endowment, then $\varphi(q, y, \alpha) = f(e(q, y) - \alpha)$. When α represents the emission standard, the compliance cost is $\varphi(q, y, \alpha) = \hat{C}(q, y, \alpha) - c(q)$, where $\hat{C}(\cdot)$ is the cost function under the given standard α.

(iii) $\gamma'(0) < -\delta\varphi_y(q, 0, \alpha) \ \forall\, q > 0, \alpha > 0,$ where $\delta \in (0,1)$ is the discount

factor[48].

For all $\alpha > 0$, assume $\lim_{q\to 0} P(Q) > p_m(0, \alpha)$, where $p_m(y, \alpha)$ is the minimum

average cost at any period. The industry equilibrium given the regulation level α consists of

the following:

(i) some measurable set of firms, $S(\underline{\tau}, \bar{\tau})$, for firms who enter the industry at

period-$\underline{\tau}$ and leave at period-$\bar{\tau}$, with $1 \le \underline{\tau} \le \bar{\tau} \le T$.

(ii) an integrable output and capital investment profile:

$$\{q_t(i), x_t(i), t = \underline{\tau}, \cdots, \bar{\tau}\} \ \forall\, i \in S(\underline{\tau}, \bar{\tau}).$$

(iii) price vector $p = \{p_1, \cdots, p_T\}$ such that

    a. market clears at every period-t: $D(p_t) = Q_t$ where $Q_t = \int_{S_t} q_t(i)\,di$,

    where S_t is the set of firms that are active at period-t.

    b. the output and capital investment profile

    $\{q_t(i), x_t(i), t = \underline{\tau}, \cdots, \bar{\tau}\} \ \forall\, i \in S(\underline{\tau}, \bar{\tau})$ solves firm-i's profit maximization

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[48] Note that $-\varphi_y$ is the marginal reduction of compliance cost attributed to increased capital. This assumption guarantees strictly positive investment benefit under some regulation $\alpha > 0$ for firms who stay in the industry for more than 1 period.

problem when the number of firms $n(\underline{\tau}, \bar{\tau}) > 0$. [49]

   c.   free entry condition is satisfied: $\Pi(p, \alpha, \underline{\tau}, \bar{\tau}) \begin{array}{l} = 0 \text{ if } n(\underline{\tau}, \bar{\tau}) > 0 \\ \leq 0 \quad \text{otherwise.} \end{array}$

It can then be shown that (as in Petrakis and Roy's paper, 1999), for all $\alpha > 0$, there exists an industry equilibrium, which happens to be the restricted social optimum.[50] Note that if there is no regulation, then assumption (i) ensures the compliance cost is zero, and thus the price path is stationary in the sense that $p_1 = p_2 = \cdots = p_T$, and thus there is no change in industry size over time, i.e. $n_1 = \cdots = n_T$. For positive regulation $\alpha > 0$ and when capital investment can change the variable cost of compliance, i.e., $\varphi_{qy} < 0$, the price path, as well as the industry size is no longer stationary, since the capital investment may ultimately change firms' marginal cost, and thus create the possibility of turnover. Then as in Petrakis and Roy (1999), they found the price path was strictly decreasing if $\varphi_{qy} < 0$, specifically, $p_1 > p_T$. It can also be proved that at equilibrium, there will be no entry after the initial period (inferred from the free-entry condition), and firms who exit earlier (before time T) are those with lower

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[49] For a firm that enters at period-$\underline{\tau}$ and leaves at period-$\bar{\tau}$, its discounted profit sum is:

$\Pi(p, \alpha, \underline{\tau}, \bar{\tau}) = \max_{(q_t, x_t) \geq 0} \sum_{t=\underline{\tau}}^{\bar{\tau}} \delta^{t-\underline{\tau}} [p_t q_t - c(q_t) - \varphi(q_t, y_t, \alpha) - \gamma(x_t)]$, where $y_t = \sum_{\tau=\underline{\tau}}^{t-1} x_\tau$, $t > \underline{\tau}$ and $y_{\underline{\tau}} = 0$.

[50] Since there's no social damage function for pollution, the equilibrium is socially optimal in a restricted sense.

accumulated investment, higher compliance cost and which are smaller in size.[51] The implication of these findings is that, environmental regulations can endogenously create heterogeneity in compliance cost and size dispersion of firms, by creating differences in capital investment and planned survival of firms.

Using a 2-period example, Sengupta identifies three effects of increased regulation (i.e. higher α) on the evolution of the industry along the time path:

(i)   For any given profile of investment, a higher level of regulation increases the cost structure of the industry, which in turn increases the equilibrium price and decreases the total quantity sold. This creates a downward pressure on the industry size in the last (second) period.

(ii)   For any given profile of investment, a higher level of regulation shifts both the average cost and the effective marginal cost upward, which alters the optimal scale of a firm directly. If the average cost curve shifts to the left while moving up, the optimal scale decreases. If the decrease in the optimal scale is more than the decrease in the total industry output, then the number of firms tends to increase with a higher level of regulation, and vice versa. That is, whether the number of firms rises or falls depends on the nature and extent of changes in the optimal scale of individual firms.

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[51] Sengupta identifies the sufficient condition for exit to occur as: $\frac{D(p_m(y,\alpha))}{q_m(y,\alpha)} < \frac{D(p_m(0,\alpha))}{q_m(0,\alpha)}$ $\forall$ $y > 0$, where $D(\cdot)$ is the quantity demanded, $p_m(y,\alpha)$ is the minimal average cost and $q_m(y,\alpha)$ the corresponding minimal efficient scale for a typical firm, whose capital stock is y. That is, if the minimal efficient scale $q_m(\cdot)$ expands too fast with investment relative to the expansion of total quantity resulting from fall in prices over time, there must be some shake-out.

(iii) Increase in regulation may increase cost-reducing investment. If this happens, there is an expansion in the optimal scale of an individual firm, which then tends to reduce the size of the industry.

Consequently, the net effect of higher level of regulation depends on how individual firm's optimal scale changes (effect (ii) and (iii)). By assumption, $\varphi_{yq} \leq 0$, i.e. the investment is more effective in reducing compliance cost at a higher level of output, implying investment reduces the marginal cost of production. The complementarity between investment and regulation thus determines the extent to which higher regulation creates an incentive for more investment, and in turn, the reduction of marginal cost and the expansion in the optimal scale of individual firms. If effect (iii), which is generated by cost-reducing investment, is strong and the marginal cost of firms falls sharply with investment, then more stringent regulation leads to higher shake-out.

From the numerical example, Sengupta shows that the impact of raising environmental regulation on an industry may be delayed.[52] From the perspective of a social planner, he/she may want a large pool of firms in the initial period to bring down the industry's total cost (if the marginal cost curve is steep). Then over time as firms invest to reduce future compliance costs, the effective marginal cost of an individual firm becomes flatter, and its optimal scale expands. Then it is no longer necessary for the social planner to keep the large industry size which may incur large fixed cost. Empirically, this model can somewhat explain the mixed

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[52] Sengupta also specifies the sufficient conditions for higher shake-out in response to higher level of regulation. From the numerical example of a 2-period model, it can be concluded that higher level of regulation may correspond to a larger industry size in the 1st period, whereas the number of firms in the 2nd period is always smaller. That is, the industry size is time-dependent per se. Larger industry size in the 1st period than smaller size in the 2nd period means there is a greater number of firms that exit the industry.

evidence of raised regulation on the industry evolution in terms of size-distribution, investment behavior, and entry-exit of firms.

CHAPTER 3: MONITORING COST, PENALTY AND RISK AVERSE FIRMS

1. Introduction

The effect of fines and risk attitudes in the permit market is still little explored to date. While some crime economics literature has suggested the fine should be as large as possible to save enforcement cost, it is unclear how this statement would hold under a tradable permit system with risk averse agents. In fact, as pointed out in much of the crime literatures (Polinsky and Shavell [1979], Cameron [1988], Garoupa [1997]), when agents are risk averse, detection probability and the penalty are no longer perfect substitutes. Moreover, there are models in which agents can engage in avoidance activities such as bribes, lobbying, etc, suggesting that in some cases, the detection probability and fines may be complements (Garoupa [2001], Langlais [2008]).

Since the paper by Malik (1990), however, there have been a sizable number of papers which discuss the enforcement effectiveness in the permit markets, and whether compliance affects social welfare (Keeler [1991], Stranlund and Dhanda [1999], Malik [2002], Mrozek and Keeler [2004]). Some work has been done on firms' risk attitude toward regulations. Ben-David et al. (2000) used a model with risk averse firms to address the uncertainty in the pollution permit market, where the permit price may be stochastic. Ozanne and White. (2008) looked at the interactions between risk averse firms and the regulator via contract theory. More recently, Stranlund (2008) re-examined Malik's noncompliance model with risk averse

agents[53]. The issue of penalty and monitoring efforts, nonetheless, in the pollution market were less explored until recent. Arguedas' (2008) model explored firms' compliance decisions under pollution standard cap, and the government's optimal policy on monitoring. Bontems and Rotillon (2008) added "involuntary violation" into the model in which they assessed agents' compliance behavior under a pollution standard, and suggested that when there were involuntary violations, increasing the audit probability could in fact lead to lower compliance rate.

As suggested in the crime literature, one possible explanation for bounded penalty is that there exists some probability of being wrongly convicted and fined, such as random error coming from either the regulator's erroneous measurement or from the production process (e.g., stochastic output).[54] In that case, setting the fine to its maximum could be too much of a deterrence for any production to exist. Another possibility is that, when the firms are risk averse then, as shown in the crime literature, it may no longer be optimal for a maximal penalty and a minimal catching probability. As such, in this chapter we develop a model addressing both the issues of uncertainty and bounds on fines in permit markets. By examining relations of the monitoring probability and penalty structure when the agents are risk averse, we characterize the equilibrium conditions and their implications on policies under this structure.

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[53] In both Ozanne et al. and Stranlund's models, the only source of uncertainty comes from the regulator's audit probability.

[54] Mrozek and Keeler (2004) adopted the involuntary violation idea by assuming the emission is stochastic. Unfortunately, in their 2-period, 2-firm model, they were unclear about the timeline of the enforcement-compliance game and the trading process. The use of the same permit price notation in both periods is also misleading, resulting in credibility issue about the conclusions.

We first compare the pollution permit demand for risk neutral and risk averse firms, and demonstrate that the demand for risk averse firms is higher. As there is uncertainty about being in compliance or not, the permits serve as a hedge product for the firm.

Next we proceed to investigate the impact of audit probability and the penalty on the firms' behavior and the permit market equilibrium. We show that, even with the possibility of being wrongly convicted, the audit probability and the penalty are still perfect substitutes when the firms are risk neutral. Nevertheless, under a particular type of risk aversion, namely CARA, firms react to the proportional change in audit probability and the penalty (while holding the expected penalty unchanged) by demanding more permits, as risk averse agents usually react more to the potential loss (compared to potential gains).

Finally we conclude the paper by looking at the permit market equilibrium conditions and the implications for regulators' policy design under this model structure.

2. The Model

The model is set up with n heterogeneous risk averse firms[55], differing in terms of the cost function. Let i denote an individual firm's subscript. Each firm's production cost depends on its production quantity q and emission e: $c_i(q_i; e_i)$, where c_i is increasing in q but decreasing in e; the marginal benefit from emission in cost-saving is decreasing, i.e. $c_{ee} > 0$, and marginal cost for production is increasing with $c_{qq} > 0$. We further assume that $c_{eq} < 0$.

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[55] In the long run, n is endogenous as firms can enter and exit the industry under free-entry condition. This issue is addressed in Chapter 4.

The regulator announces its inspection probability π and an increasing penalty schedule $G(v)$ before the firms take actions on production and emissions, where $G(0) = 0$ and $G', G'' > 0$, where v is the *measured violation*.[56]   Similar to Mrozek and Keeler, we assume there is uncertainty in emission, which could be caused by measurement error by the regulator. Specifically, the measured emission is a function of the firm's actual emission e and an error term λ,[57] which follows some random distribution F. The firm's risk aversion is captured in its utility function $u(R)$, with $u' > 0$ and $u'' < 0$, where R is the firm's profit.[58] The government issues a total amount, L, of permits for firms to purchase freely (i.e. no transaction cost) in a competitive permit market. Let p and t be the equilibrium permit and product prices, respectively, and let ℓ_i denote firm-i's permit holding. The following summarizes the utility-maximizing problem for any firm-i (I omit the firm-subscript for simplicity):

(1) $$\max_{\ell, e, q} E(u) = \left[1 - \pi + \pi F(\hat{\lambda}) \right] u(R_0) + \pi \int_{\hat{\lambda}}^{\infty} u(R_1) f(\lambda) d\lambda ,$$

where

$$R_0 = tq - c(q; e) - p\ell$$
$$R_1 = R_0 - G(e - \ell + \lambda) = tq - c(q; e) - p\ell - G(e - \ell + \lambda)$$
$$\hat{\lambda} = \ell - e.$$

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[56] Here defining $G(0) = 0$ avoids the discontinuity issue.

[57] For simplicity, we assume the measurement error is additive, i.e., the measured emission for any firm i is $e_i + \lambda_i$, and the measured violation is simply $v_i = e_i + \lambda_i - \ell_i$. Note that λ is the exogenous uncertainty, whereas the audit frequency (another uncertain factor) is controlled by the government, who can (in theory) adjust the frequency optimally to maximize social welfare.

[58] Here R includes a firm's wealth (e.g., endowment or transfers from the government.)

The first bracket of the maximization problem is the probability of not being audited, plus the probability that the firm is audited and recognized as compliant by the government; the second term is the expected utility from being audited and recognized as noncompliant. Note that when the actual violation is equal to $\hat{\lambda}$, there is no "measured violation" per se. R_0 and R_1 are firms' profit from being "recognized compliant" and "recognized noncompliant," respectively.

The FOC w.r.t the problem are:

$$(\qquad\qquad 2 \qquad\qquad)$$

$$\frac{\partial E(u)}{\partial \ell} = -\left[1 - \pi + \pi F(\hat{\lambda})\right] p u'(R_0) + \pi \int_{\hat{\lambda}}^{\infty} u'(R_1) G' f(\lambda) d\lambda - p\pi \int_{\hat{\lambda}}^{\infty} u'(R_1) f(\lambda) d\lambda = 0$$

$$(\qquad\qquad 3 \qquad\qquad)$$

$$\frac{\partial E(u)}{\partial e} = -\left[1 - \pi + \pi F(\hat{\lambda})\right] c_e u'(R_0) - \pi \int_{\hat{\lambda}}^{\infty} u'(R_1) G' f(\lambda) d\lambda - c_e \pi \int_{\hat{\lambda}}^{\infty} u'(R_1) f(\lambda) d\lambda = 0$$

(4) $$\frac{\partial E(u)}{\partial q} = (t - C_q)\left\{ \left[1 - \pi + \pi F(\hat{\lambda})\right] u'(R_0) + \pi \int_{\hat{\lambda}}^{\infty} u'(R_1) f(\lambda) d\lambda \right\} = 0$$

The usual result, that the equilibrium permit price equals the marginal cost saved by polluting holds, and the optimal condition for production are straightforward from the FOC [59]:

(5) $\qquad p = -c_e$

(6) $\qquad t = c_q$

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[59] Keep in mind that in the multiplicative case, i.e., when the measured emission is λe, risk aversion would affect the optimal emission choice and these results would not hold.

It is then apparent that the firm's emission and production decisions depends solely on the given permit and product prices, i.e., $e(t, p)$ and $q(t, p)$, and the shape of its cost function (a result directly from equation (5) and (6)); and since the firm does not engage in any other pollution abatement activities in the model, the usual market equilibrium condition for production holds. The only thing that is effected by the firm's risk attitude, though, is its choice of permit holding. The intuition is: because there is an uncertainty of being in violation or not, firms may choose to hedge by buying more permits. This is the same as in the forward market, where the output is determined by the futures price, and risk attitude affects the number of future contracts held.

We then begin the analysis by first assessing how the risk attitude affects the firms' permit demand. Consider the case where the firm is risk neutral, i.e., its utility function is linear. Equation (2) can then be rewritten as:

$$\frac{\partial E(u)}{\partial \ell} = -\left[1 - \pi + \pi F(\hat{\lambda})\right] + \left[\pi \int_{\hat{\lambda}}^{\infty}\left(\frac{G'}{p} - 1\right)f(\lambda)d\lambda\right] = 0$$

$\Rightarrow$

(2a) $\qquad \int_{\hat{\lambda}}^{\infty}\left(\frac{G'}{p} - 1\right)f(\lambda)d\lambda = \frac{1 - \pi + \pi F(\hat{\lambda})}{\pi}$ [60]

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[60] (2a) can also be written as: $\pi \int_{\hat{\lambda}}^{\infty} G' f(\lambda)d\lambda = p$. That is, when firms are risk neutral, their decision rule on the amount of permits to hold is equating the expected penalty (given that the firm is fined) and the permit price.

Let ℓ^N be the solution to equation (2a). The following proposition states the effect of risk aversion on permit demand:

Proposition 1.

The permit demand for risk averse firms are higher than risk neutral ones.

Proof.

By the SOSC, $\dfrac{\partial^2 E(u)}{\partial \ell^2} < 0$; so we need to prove that $\left.\dfrac{\partial E(u)}{\partial \ell}\right|_{\ell^N} > 0$. To show this, first rewrite equation (2) as:

(2b)
$$\frac{\partial E(u)}{\partial \ell} = -\left[1 - \pi + \pi F(\hat{\lambda})\right] + \left[\pi \int_{\hat{\lambda}}^{\infty}\left(\frac{u'(R_1)}{u'(R_0)}\right)\left(\frac{G'}{p} - 1\right)f(\lambda)d\lambda\right]$$

$$= \int_{\hat{\lambda}}^{\infty}\left(\frac{u'(R_1)}{u'(R_0)}\right)\left(\frac{G'}{p} - 1\right)f(\lambda)d\lambda - \frac{1 - \pi + \pi F(\hat{\lambda})}{\pi}$$

Then, it is sufficient to show that :

(2c)
$$\int_{\hat{\lambda}}^{\infty}\left(\frac{u'(R_1)}{u'(R_0)}\right)\left(\frac{G'}{p} - 1\right)f(\lambda)d\lambda > \int_{\hat{\lambda}}^{\infty}\left(\frac{G'}{p} - 1\right)f(\lambda)d\lambda .$$

(i) First, $\dfrac{u'(R_1)}{u'(R_0)} > 1$ since $u'' < 0$ for risk aversion, thus if $\dfrac{G'(0)}{p} > 1$, it is clear that the inequality (2c) holds.

(ii) Consider the case in which $\dfrac{G'(0)}{p} < 1$, then for (2a) to hold, there exists some $\tilde{\lambda}$ such

that $\dfrac{G'(\lambda)}{p} \geq 1 \;\; \forall \, \lambda \geq \tilde{\lambda}$, and $\dfrac{G'(\lambda)}{p} < 1$ otherwise, since $G'' > 0$ and

$\displaystyle\int_{\hat{\lambda}}^{\infty} \left(\dfrac{G'}{p} - 1 \right) f(\lambda) d\lambda > 0 \; {}^{61}.$

Define $D \equiv \dfrac{u'(\tilde{R}_1)}{u'(R_0)} = \dfrac{u'\big(R_0 - G(\tilde{\lambda})\big)}{u'(R_0)} > 1$ (since $u'' < 0$). Then:

$$\int_{\hat{\lambda}}^{\infty} \left(\dfrac{u'(R_1)}{u'(R_0)} \right) \left(\dfrac{G'}{p} - 1 \right) f(\lambda) d\lambda = \int_{\hat{\lambda}}^{\infty} \left(\dfrac{u'(R_1)}{u'(R_0)} - D \right) \left(\dfrac{G'}{p} - 1 \right) f(\lambda) d\lambda + D \int_{\hat{\lambda}}^{\infty} \left(\dfrac{G'}{p} - 1 \right) f(\lambda) d\lambda .$$

The first term on the right hand side is always positive because $\dfrac{u'(R_1)}{u'(R_0)} - D \geq 0 \; \forall \, \lambda \geq \tilde{\lambda}$, and $\dfrac{u'(R_1)}{u'(R_0)} - D < 0$ otherwise, while $\dfrac{G'(\lambda)}{p} \geq 1 \; \forall \, \lambda \geq \tilde{\lambda}$ and

$\dfrac{G'(\lambda)}{p} < 1 \; \forall \, \lambda < \tilde{\lambda}$.

The product of $\left(\dfrac{u'(R_1)}{u'(R_0)} - D \right)$ and $\left(\dfrac{G'}{p} - 1 \right)$ is then always positive. As a result, the right

hand side is always greater than $\displaystyle\int_{\hat{\lambda}}^{\infty} \left(\dfrac{G'}{p} - 1 \right) f(\lambda) d\lambda$ since $D > 1$.

That is, $\displaystyle\int_{\hat{\lambda}}^{\infty} \left(\dfrac{u'(R_1)}{u'(R_0)} \right) \left(\dfrac{G'}{p} - 1 \right) f(\lambda) d\lambda > \int_{\hat{\lambda}}^{\infty} \left(\dfrac{G'}{p} - 1 \right) f(\lambda) d\lambda$.

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[61] This is a necessary condition for any positive permit trade. That is, the conditional expected marginal penalty is greater than the permit price: $E(G' \,|\, \lambda > \hat{\lambda}) > p$.

Q.E.D. ∎

Besides the permit demand, risk attitude also plays a role in the neutrality between the penalty function and the audit probability. At the optimum, $E(u)$ is a function of $(q^*, \ell^*, e^*; t, p, \pi, G)$. Now suppose the audit probability and the penalty function $\{\pi, G\}$ becomes $\left\{\frac{\pi}{\delta}, \delta G\right\}$ (with $\delta \geq 1$) such that, given the same firm behavior, the expected fine is unchanged. The following proposition states the neutrality in the case where firms are risk neutral.

Proposition 2.

When firms are risk neutral, the proportional change in the audit probability and the penalty function such that the expected fine remains the same, will not effect the firms' optimal decision.

Proof.

To see this, note that equation (1) can be rewritten as:

(1a) $$E(u) = u(R_0) + \pi \int_{\hat{\lambda}}^{\infty} \left[u(R_1) - u(R_0) \right] f(\lambda) d\lambda.$$

Under risk neutrality, the utility function is linear. Suppose the utility function is: $u(R) = \alpha + \beta R$:

(1b) $$E(u) = u(R_0) + \pi \int_{\hat{\lambda}}^{\infty} \left[-\beta G \right] f(\lambda) d\lambda.$$

Then at optimum, equation (1b) can be rewritten in terms of $\left\{\frac{\pi}{\delta}, \delta G\right\}$:

$$E(u^*) = u(R_0) + \frac{\pi}{\delta}\int_{\hat{\lambda}}^{\infty}[-\delta\beta G]f(\lambda)d\lambda = u(R_0) + \pi\int_{\hat{\lambda}}^{\infty}[-\beta G]f(\lambda)d\lambda,$$

which is exactly the same as the original (1b) under $\{\pi, G\}$.

Q.E.D. ∎

Proposition 2 implies that, when firms are risk neutral and when monitoring/auditing is costly, regardless of whether there exists involuntary violation or not, regulators have strong incentive to set the audit probability close to zero, and make the fine as high as possible, provided there's no economic cost or deadweight loss due to the penalty. This finding is identical to what many papers in the crime literatures have suggested: when agents are risk neutral, the optimal detection rate is at its minimum while the optimal fine is the maximum fine[62]. This is also a major point that many papers on the permit markets have overlooked.

The question then becomes whether this neutrality would hold under risk aversion. We begin by proving the following corollary for the optimal $E(u^*)$.

Proposition 3.

The maximized expected utility for risk averse firms decreases when there is a proportional decrease in audit probability and an increase in fine (such that the expected penalty remains the same), i.e., as $\{\pi, G\} \to \left\{\dfrac{\pi}{\delta}, \delta G\right\}$.

---

[62] Due to bankruptcy, the maximum penalty is constrained by a firm's wealth.

Proof.

Rewrite equation (1a) in terms of $\left\{\dfrac{\pi}{\delta}, \delta G\right\}$:

(1c) $$E(u^*) = u(R_0) + \frac{\pi}{\delta} \int_{\hat{\lambda}}^{\infty} \left[u(R_0 - \delta G) - u(R_0) \right] f(\lambda) d\lambda$$

Differentiate (1c) w.r.t. δ, and invoke the envelope theorem:

$$\frac{\partial E(u^*)}{\partial \delta} = \frac{\pi}{\delta^2} \int_{\hat{\lambda}}^{\infty} \left[u(R_0) - u(R_0 - \delta G) - \delta G u'(R_0 - \delta G) \right] f(\lambda) d\lambda$$

Use Taylor's expansion, $u(R_0)$ can be expressed as:

$$u(R_0) = u(R_0 - \delta \tilde{G}) + \delta \tilde{G} u'(R_0 - \delta \tilde{G}) + \frac{u''(y)(\delta \tilde{G})^2}{2} \quad \forall \text{ some } y \in (R_0 - \delta \tilde{G}, R_0), \tilde{G} \equiv G(\lambda - \tilde{\lambda}).$$

Substitute that into $\dfrac{\partial E(u^*)}{\partial \delta}$, and note that with risk aversion, $u'' < 0$, $\dfrac{\partial E(u^*)}{\partial \delta} < 0$

Q.E.D. ∎

The above proposition proves that, under risk aversion, the neutrality, in terms of expected utility, between audit probability and fines no longer holds. Next we turn our attention to how this proportional change effects the permit demand, and the consequent equilibrium in the permit market. Because risk aversion does complicate things, we adopt a specific type of risk aversion, i.e., the constant absolute risk aversion, or CARA, to make tractable conclusions under risk aversion in the following analysis.

Proposition 4.

A proportional decrease of audit probability and an increase of fine such that the expected penalty remains unchanged, will lead to an increase in permit demand when the firm exhibits CARA.

Proof.

Let

$$K(\delta) \equiv \frac{\partial E(u)}{\partial \ell} = -\left[1 - \frac{\pi}{\delta} + \frac{\pi}{\delta}F(\hat{\lambda})\right]pu'(R_0) + \pi\int_{\hat{\lambda}}^{\infty} u'(R_0 - \delta G)G'f(\lambda)d\lambda - \frac{\pi}{\delta}p\int_{\hat{\lambda}}^{\infty} u'(R_0 - \delta G)f(\lambda)d\lambda = 0$$

.

That is, the FOC for ℓ with $\left\{\frac{\pi}{\delta}, \delta G\right\}$. Let $\tilde{M} = 1 - \frac{\pi}{\delta} + \frac{\pi}{\delta}F(\hat{\lambda})$ and $\tilde{R}_1 = R_0 - \delta G$:

$$\frac{\partial K}{\partial \delta} = \left(-\frac{\pi}{\delta^2} + \frac{\pi}{\delta^2}F(\hat{\lambda})\right)pu'(R_0) - \pi\int_{\hat{\lambda}}^{\infty} u''(\tilde{R}_1)G \cdot G'f(\lambda)d\lambda + \frac{\pi}{\delta^2}p\int_{\hat{\lambda}}^{\infty} u'(\tilde{R}_1)f(\lambda)d\lambda + \frac{\pi}{\delta}p\int_{\hat{\lambda}}^{\infty} u''(\tilde{R}_1)Gf(\lambda)d\lambda$$

$$= -\frac{\pi}{\delta^2}p\int_{\hat{\lambda}}^{\infty}\left[u'(R_0) - u'(\tilde{R}_1)\right]f(\lambda)d\lambda - \pi\int_{\hat{\lambda}}^{\infty} u''(\tilde{R}_1)G\left(G' - \frac{p}{\delta}\right)f(\lambda)d\lambda$$

Moreover, CARA implies $u'' = -\phi u'$, where $\phi > 0$ is the absolute risk aversion coefficient. Using this, the above equation can be written as:

(7) $$\frac{\partial K}{\partial \delta} = -\frac{\pi}{\delta^2}p\int_{\hat{\lambda}}^{\infty}\left[u'(R_0) - u'(\tilde{R}_1)\right]f(\lambda)d\lambda + \pi\phi\int_{\hat{\lambda}}^{\infty} u'(\tilde{R}_1)G\left(G' - \frac{p}{\delta}\right)f(\lambda)d\lambda .$$

The first term is always positive since $u'' < 0$ (risk averse) and $R_0 > \tilde{R}_1 \ \forall \ \lambda > \hat{\lambda}$. Then we use the technique in the proof of proposition 1 to sign the second term:

(i) If $G' > \frac{p}{\delta} \ \forall \ \lambda > \hat{\lambda}$, then the second term is everywhere positive.

(ii) Otherwise, there exists some $\tilde{\lambda}$ such that $G' \gtreqless \frac{p}{\delta} \ \forall \ \lambda \gtreqless \tilde{\lambda}$, since $G'' > 0$. Define

$\tilde{G} = G(\tilde{\lambda})$, then:

$$\int_{\hat{\lambda}}^{\infty} u'(\tilde{R}_1)G\left(G' - \frac{p}{\delta}\right)f(\lambda)d\lambda = \int_{\hat{\lambda}}^{\infty} u'(\tilde{R}_1)(G - \tilde{G})\left(G' - \frac{p}{\delta}\right)f(\lambda)d\lambda + \tilde{G}\int_{\hat{\lambda}}^{\infty} u'(\tilde{R}_1)\left(G' - \frac{p}{\delta}\right)f(\lambda)d\lambda$$

,

where the first term is positive since $G \underset{<}{\overset{>}{=}} G'$ whenever $G' \underset{<}{\overset{>}{=}} \frac{p}{\delta}$, and the second term is

positive from the FOC where $K(\delta) = 0$:

$$\int_{\hat{\lambda}}^{\infty} u'(\tilde{R}_1)G' f(\lambda)d\lambda - \frac{p}{\delta}\int_{\hat{\lambda}}^{\infty} u'(\tilde{R}_1)f(\lambda)d\lambda = \frac{\tilde{M}pu'(R_0)}{\pi} > 0.$$

As a result, the second term in (7) is always positive, which leads to $\dfrac{\partial K}{\partial \delta} > 0$.

Then, since $\dfrac{\partial K(\ell, \delta)}{\partial \ell} < 0$ by the SOSC, and $\dfrac{\partial K(\ell, \delta)}{\partial \delta} > 0$, then $\dfrac{\partial \ell}{\partial \delta} = -\dfrac{\partial K}{\partial \delta} / \dfrac{\partial K}{\partial \ell} > 0$.

Q.E.D. ∎

The above propositions have demonstrated the impact of risk aversion on the response of

firms to regulators' instrument manipulation. We next derive the comparative statics in order

to look into the effects on permit markets. What we would like to know is how the

equilibrium prices, emissions and output would change given δ.

Comparative Statics

First, for an individual firm, the effect of permit price on emission and production can be

readily derived from equation (5) and (6):

$$\begin{pmatrix} c_{ee} & c_{eq} \\ c_{eq} & c_{qq} \end{pmatrix} \begin{pmatrix} de \\ dq \end{pmatrix} = \begin{pmatrix} -dp \\ dt \end{pmatrix}$$

Then by SOSC, $\Delta \equiv c_{ee}c_{qq} - c_{eq}^2 > 0$. Using Cramer's rule, we get:

$$\begin{pmatrix} de \\ dq \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} -c_{qq}dp - c_{eq}dt \\ c_{eq}dp + c_{ee}dt \end{pmatrix}.$$

Thus, for any individual firm:

(8) $$\frac{\partial e}{\partial p} = \frac{-c_{qq}}{\Delta} < 0$$

and

(9) $$\frac{\partial q}{\partial p} = \frac{c_{eq}}{\Delta} < 0 \ [63].$$

That is, as permit price goes up, it is relatively less expensive for the firm to simply reduce its emission and/or production than buying (the same amount of) permits. Consequently, the aggregate emission and production both decrease as the permit price rises since $\sum_{i=1}^{n} \frac{\partial e_i}{\partial p} < 0$ and $\sum_{i=1}^{n} \frac{\partial q_i}{\partial p} < 0$.

Next, because it is not easy to derive the comparative statics for permit demand without further assumption on the risk aversion type, we adopt the special case CARA again in the following analysis. Proposition 5 describes a downward-sloping permit demand.

Proposition 5.

As permit price rises, aggregate permit demand of firms with CARA decreases.

Proof.

Define the FOC as:

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[63] Recall that $c_{qq} > 0$ and $c_{eq} < 0$.

$$(10)$$

$$K(\ell,e,q;t,p,\pi,G;F) \equiv \left. \frac{\partial E(u)}{\partial \ell} \right|_{\ell^*,e^*,q^*} = -pu'(R_0) + \pi \int_{\hat{\lambda}}^{\infty} \left[u'(R_1)G' - p\left(u'(R_1) - u'(R_0)\right) \right] f(\lambda) d\lambda = 0.$$

Then, $\dfrac{\partial \ell}{\partial p} = -\left(\dfrac{\partial K}{\partial e}\right)\left(\dfrac{\partial e}{\partial p}\right) \Big/ \left(\dfrac{\partial K}{\partial \ell}\right) - \left(\dfrac{\partial K}{\partial p}\right) \Big/ \left(\dfrac{\partial K}{\partial \ell}\right)$ [64].

(i) First note that:

$$\frac{\partial K}{\partial e} = -p^2 u''(R_0) + \pi u'(R_0)G'(0) - \pi \int_{\hat{\lambda}}^{\infty} u''(R_1)(G'-p)^2 f(\lambda)d\lambda + \pi \int_{\hat{\lambda}}^{\infty} u'(R_1)G'' f(\lambda)d\lambda - p^2\pi \int_{\hat{\lambda}}^{\infty} u''(R_0)f(\lambda)$$

Then by risk aversion ($u'' < 0$), $\dfrac{\partial K}{\partial e} > 0$.

(ii) From (10):

$$\frac{\partial K}{\partial p} = -u'(R_0) - \pi \int_{\hat{\lambda}}^{\infty} \left[u'(R_1) - u'(R_0)\right] f(\lambda)d\lambda$$

$$+ p\ell u''(R_0) - \pi p\ell \int_{\hat{\lambda}}^{\infty} u''(R_0)f(\lambda)d\lambda - \pi\ell \int_{\hat{\lambda}}^{\infty} u''(R_1)(G'-p)f(\lambda)d\lambda$$

Using equation (2): $\pi \int_{\hat{\lambda}}^{\infty} u'(R_1)(G'-p)f(\lambda)d\lambda = 1 - \pi + \pi F(\hat{\lambda})$ and the fact that

$u'' = -\phi u'$ by CARA, $\dfrac{\partial K}{\partial p}$ can be reduced to:

$$\frac{\partial K}{\partial p} = -u'(R_0) - \pi \int_{\hat{\lambda}}^{\infty} \left[u'(R_1) - u'(R_0)\right] f(\lambda)d\lambda < 0,$$

since the second term is everywhere negative by risk aversion ($R_0 > R_1$).

---

[64] Note that $\dfrac{\partial K}{\partial q} = 0$, since $\dfrac{\partial K}{\partial q} = -pu''(R_0)(t - C_q) + \pi(t - C_q)\int_{\hat{\lambda}}^{\infty} \left[u''(R_1)G' - p\left(u''(R_1) - u''(R_0)\right)\right]f(\lambda)d\lambda$,

and at optimum, $t = C_q$.

Combine the above results with (8), and $\dfrac{\partial K}{\partial \ell} < 0$, we get: $\dfrac{\partial \ell}{\partial p} < 0$ for any individual

firm. And on aggregation, $\displaystyle\sum_{i=1}^{n} \dfrac{\partial \ell_i}{\partial p} < 0$

Q.E.D. ∎

Note that the effect of permit price on a firm's violation decision, however, is not clear

from simply (8) and Proposition 5. The final impact on violation still depends on the

magnitude of changes in emission and in permit holding.[65]

Finally, we look at the effect of audit probability and fine on permit demand. Using (10),

we can derive that $\dfrac{\partial \ell}{\partial \pi} = -\left(\dfrac{\partial K}{\partial \pi}\right) \Big/ \left(\dfrac{\partial K}{\partial \ell}\right) > 0$;[66] and with the CARA assumption, using similar

argument in the proof of Proposition 4, we can also prove that, when the penalty *G* alone

increases in scale, e.g., $G \rightarrow \delta G$, ceteris paribus, the permit demand increases

$$\frac{\partial \ell}{\partial \delta} = -\left(\frac{\partial K}{\partial \delta}\right) \Big/ \left(\frac{\partial K}{\partial \ell}\right) > 0 \,.$$

The above analyses demonstrate that, when firms are risk averse with CARA, the

aggregate demand for permit is downward-sloping; with fixed permit supply, the equilibrium

is determined by the demand. Then combining with the result in Proposition 4, as $\{\pi, G\}$

---

[65] Recall that $v = e + \lambda - \ell$; a firm's actual violation is simply $\tilde{v} = e - \ell \Rightarrow \dfrac{\partial \tilde{v}}{\partial p} = \dfrac{\partial e}{\partial p} - \dfrac{\partial \ell}{\partial p}$.

[66] From (10), $\dfrac{\partial K}{\partial \pi} = \displaystyle\int_{\hat{\lambda}}^{\infty} \left[u'(R_1)G' - p\left(u'(R_1) - u'(R_0)\right) \right] f(\lambda)\,d\lambda = \dfrac{pu'(R_0)}{\pi} > 0$. Use equation (2) to get
the second equality. Note that this relationship does not require further assumption (e.g., CARA) on risk
aversion type.

becomes $\left\{\frac{\pi}{\delta}, \delta G\right\}$, the permit price will go up as the demand rises, and the total emission will

go down.

Another remark from the analysis is, since the firm's permit demand is increasing in both the audit probability and the fine, the fact that the firm exhibits rising demand when there's a proportional decrease in the audit probability and an increase in fine (holding expected penalty unchanged), suggests that the permit demand is more sensitive to the severity of punishment and less so to the catching probability. This is a consistent finding with empirical evidence on risk attitude (Neilson and Winter [1997]).

Permit Market Equilibrium

From the FOC equations (5) to (7), it can be derived that the optimal permit demand is a function of the parameters: $\ell_i^* = \ell_i(p, t; \pi, G, F)$. The market clearing condition requires that:

(11) $$\sum_{i=1}^{n} \ell_i^* = \sum_{i=1}^{n} \ell_i(p, t; \pi, G, F) = L$$

where L is the permit supply. The implicit differentiation of (11) yields:

$$\frac{\partial p}{\partial L} = 1 / \sum_{i=1}^{n} \frac{\partial \ell_i}{\partial p} < 0$$

$$\frac{\partial p}{\partial \pi} = -\sum_{i=1}^{n} \frac{\partial \ell_i}{\partial \pi} / \sum_{i=1}^{n} \frac{\partial \ell_i}{\partial p} > 0$$

Then by continuity, there exists some policy set $\left\{\tilde{L}, \tilde{\pi}, \tilde{G}\right\}$ (e.g., $p^{e}(L, \frac{\pi}{\delta}, \delta G)$ where

$\frac{\partial L}{\partial \delta}\bigg|_{p^e} > 0$), such that the equilibrium permit price, and hence the level of pollution, are held

unchanged (given the number of firms) as the government adjusts the three instruments.

3. Conclusion and Policy Implication

In our model, because there is some possibility of being fined even if the firm is in compliance, the pollution permits in this case thus act like a hedge for the firms[67]: holding more permits reduces the chance of being recognized as a violator. As Proposition 1 has stated, in a market with risk averse firms, the permit demand is higher than that of risk neutral firms. Then, with further assumptions on risk aversion, i.e., CARA, we can conclude that the equilibrium permit price is higher when agents are risk averse, given a fixed permit supply. As mentioned before, this result is consistent with the hedging literature, where agents would invest more in the hedging product (in our case, firms hedge by buying more permits) to avoid possible financial loss, which lead to an increase in the demand of the hedge product, i.e., the permit, and its price.

Another important finding from our model is that, under risk neutrality, the audit probability and the severity of penalty are perfect substitutes. That is, there is no reason for a cost-minimizing government not to set the audit probability to its minimum and the penalty to

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[67] In fact, the source of the uncertainty is not a crucial factor in our model. For instance, if $e + \lambda$ is the true emission (so that e is the expected emission), everything remains the same.

its maximum[68], even with the uncertainty of being wrongfully convicted, since a proportional change in the probability of being caught and the penalty have no effects on the firms' decisions, nor the market equilibrium.

Then, as our model has concluded, Proposition 4 and 5 together offer an economic justification for making the fine bounded: since the neutrality between the audit probability and punishment no longer holds, increasing the fine and reducing the catching probability would hurt the firms by driving up the permit price, causing the firms to spend more on permit purchase. Thus, letting the fine grow unbounded is not without consequence. Yet on the other hand, considering the fact that the government actually has more than those two instruments to manipulate in the permit market, as the analysis in last section goes, from the market clearing condition, it is as well plausible for the government to decrease the audit probability (to save up monitoring cost), while increasing the penalty schedule and the total number of permits issued at the same time, so as to maintain the same market equilibrium price. Under this circumstance, though, increasing the fine is not a purely bad thing to the market.

Keep in mind that we have adopted a special case of risk aversion, i.e., CARA to derive some of the results. Under CARA the wealth effect can be purged of, and its special form makes it easy to derive the comparative statics. Those results may or may not hold with different types of risk aversion, under which we need to consider about the wealth effect, such as the initial permit endowments, and how it would interact with government's instruments.

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[68] We acknowledge there might still be some reasons other than the economic ones, e.g., legal process or moral issues, such that the government cannot (or will not) set up a penalty too extreme. These are not in the scope of our discussion.

Another note is, throughout the model, we have used additive error term in the analysis, even though it might be more realistic if the measurement error is multiplicative, i.e., $v = \lambda e - \ell$, where the error is proportional to the actual emission. With multiplicative error term, however, we might not get a clear-cut analytical solution but need to use numerical method to approximate one. In the next Chapter, we assess the case when the error term is multiplicative and present the numerical simulation results in Chapter 5.

Another possible extension to the model is also on the error term, particularly, to assess how the variance of the error term would affect the firms' behavior. Generally speaking, a risk averse firm would be worse off if the error term has a larger variance (assuming a mean-preserving spread). While the intuition implies that a firm would respond to higher variance (higher risk) by demanding more hedges (permits), a more detailed analysis is needed to see the actual impact of the second-order stochastic dominance.

CHAPTER 4: MULTIPLICATIVE ERROR TERM

In the previous chapter, we assessed the role of uncertainty in permit markets, and found that there is no neutrality between two of the government's policy instruments, namely the monitoring probability and the penalty. Because the government can adjust both instruments, in the short run, it is more efficient for government to implement the permit system by raising penalty schedule and lowering the audit probability, in order to save the implementing cost, while raising total permit amount at the same time.

Though it is analytically plausible to assume an additive measurement error term, it may not be a proper assumption in the real world application. To think of measurement error on the pollution, it may come from multiple sources: the mechanic error in measuring technique, and/or the nature of the pollutants (chemicals that are hard to monitor). If only considering the measuring technique, the assumption of additive error seems adequate; however, when the pollution itself is dispersed, treating the error as multiplicative is more realistic.

To be specific, the more interesting (and more realistic) case is that the measurement error is proportionate to the actual pollution.[69] Recall in the previous setup, the measured violation for any firm-i was:

$$v_i = e_i + \lambda_i - \ell_i,$$

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[69] As firms pollute more, the larger the measurement error will be. Note that in our model we assume the uncertainty comes from the government's measurement. But the analytical results are robust to any uncertainty sources (e.g., production shocks, weather impact, etc.)

where ℓ_i is the permit held by firm-i, e_i the actual emission, and λ_i represents the measurement error. With multiplicative error, the measured violation becomes:[70]

$$v_i = \lambda_i e_i - \ell_i.$$

We then proceed to re-examine the propositions under this structure.

The Model

As before, let R_0 be the base revenue when there's no measured violation. $c(\cdot)$ is the firm's production function, which is decreasing in emission e and increasing in production q, namely, $c_e < 0 < c_q$. We further assume $c_{ee}, c_{qq} > 0$ and $c_{eq} < 0$. The penalty function $G(\cdot)$ depends on the measured violation. Assume error $\lambda_i \overset{iid}{\sim} f(\cdot)$.

The firm's objective is to maximize its expected utility, which can be expressed as (the indicator i is dropped for simplicity):

(1) $$\xi \equiv \max_{\ell, e, q} E(u) = \left[1 - \frac{\pi}{\delta} + \frac{\pi}{\delta} F(\hat{\lambda}) \right] u(R_0) + \frac{\pi}{\delta} \int_{\hat{\lambda}}^{\infty} u(R_1) f(\lambda) d\lambda,$$ [71]

where

$$R_0 = tq - c(q;e) - p\ell$$
$$R_1 = R_0 - \delta G(\lambda e - \ell) = tq - c(q;e) - p\ell - \delta G(\lambda e - \ell)$$
$$\hat{\lambda} = \frac{\ell}{e}.$$

---

[70] Note that it is $v_i = (1 + \varepsilon_i)e_i - \ell_i$, where $\lambda_i = 1 + \varepsilon_i$. We use the expression of λ_i to keep the model easy to assess.

[71] Here we assume the policy set is $\left\{ \dfrac{\pi}{\delta}, \delta G \right\}$ with $0 < \dfrac{\pi}{\delta} \leq 1$.

The first bracket of the maximization problem is the probability of not being audited, plus the probability that the firm is audited and recognized as compliant by the government; the second term is the expected utility from being audited and recognized as noncompliant.

Note that Equation (1) can be rewritten as:

(1a) $$\xi \equiv \max E(u) = u(R_0) + \frac{\pi}{\delta} \int_{\hat{\lambda}}^{\infty} \left[u(R_1) - u(R_0) \right] f(\lambda) d\lambda$$

The FOCs are:

(2) $$\xi_q = \frac{\partial E(u)}{\partial q} = (t - c_q) \left\{ u'(R_0) + \frac{\pi}{\delta} \int_{\hat{\lambda}}^{\infty} \left[u'(R_1) - u'(R_0) \right] f(\lambda) d\lambda \right\} = 0$$

(3)

$$\xi_e = \frac{\partial E(u)}{\partial e} = -c_e \left\{ u'(R_0) + \frac{\pi}{\delta} \int_{\hat{\lambda}}^{\infty} \left[u'(R_1) - u'(R_0) \right] f(\lambda) d\lambda \right\} - \pi \int_{\hat{\lambda}}^{\infty} u'(R_1) G' \lambda f(\lambda) d\lambda = 0$$

(4)

$$\xi_\ell = \frac{\partial E(u)}{\partial \ell} = -p \left\{ u'(R_0) + \frac{\pi}{\delta} \int_{\hat{\lambda}}^{\infty} \left[u'(R_1) - u'(R_0) \right] f(\lambda) d\lambda \right\} + \pi \int_{\hat{\lambda}}^{\infty} u'(R_1) G' f(\lambda) d\lambda = 0$$

From (2) the production q^* can be solved given product price t and emission e:

(5) $$q^* = q(t, e).$$

Substituting it back to (1a), the model can be simplified down to two choice variables: $\{e, \ell\}$, and the Hessian is:

(6) $$H = \begin{bmatrix} V_{ee} & V_{e\ell} \\ V_{e\ell} & V_{\ell\ell} \end{bmatrix},$$

where V is the objective function with $q^* = q(e, t)$.

We can then establish the comparative statics for firms' choice variables.

Proposition 1.

Firm-i's permit demand is decreasing in permit price if it exhibits CARA. (Same as the additive error case.)

Proof.

For simplicity, let $z = u'(R_0) + \dfrac{\pi}{\delta} \int_{\hat{\lambda}}^{\infty} \left[u'(R_1) - u'(R_0) \right] f(\lambda) d\lambda$; note that $z > 0$ since by risk aversion, $u' > 0$ and $u'' < 0$ (and $R_0 > R_1$).

Rewrite the FOCs as (note here $c_e = c_e \left(q^*(t, e); e \right)$):

(3a) $\qquad V_e = -c_e z - \pi \int_{\hat{\lambda}}^{\infty} u'(R_1) G' \lambda f(\lambda) d\lambda = 0$

(4a) $\qquad V_\ell = -pz + \pi \int_{\hat{\lambda}}^{\infty} u'(R_1) G' f(\lambda) d\lambda = 0$

Totally differentiate the set of FOC w.r.t. p we get:

(7) $\qquad H \begin{pmatrix} \partial e / \partial p \\ \partial \ell / \partial p \end{pmatrix} = \begin{pmatrix} -V_{ep} \\ -V_{\ell p} \end{pmatrix}.$

Differentiate (3a) w.r.t p and apply CARA ($u'' = -\phi u'$), we get $V_{ep} = 0$.[72]

Adopting the same technique, we can also show that $V_{\ell p} = -z < 0$.

Then by Cramer's Rule, we get $\dfrac{\partial \ell}{\partial p} = \dfrac{-V_{ee} V_{\ell p}}{|H|} < 0$, where $|H| > 0$ and $V_{ee} < 0$ by SOSC.

Q.E.D.■

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[72] See Appendix 1.

The effects of permit price on pollution and production, however, depends on the substitutability between permit demand and emission. To see this, assume the firm's utility function is of the following CARA functional form:

(8) $\qquad u(R) = -\dfrac{\exp(-\phi R)}{\phi}, \ \phi > 0 .$

Corollary.

Assuming the firm's utility function as in (8), its permit demand and emission are complements, i.e., $V_{e\ell} > 0$.

Proof.

From (3a) and (8), we have:

(9)

$$V_{e\ell} = u'(R_0)\pi \left\{ G'(0)f\left(\hat{\lambda}\right)\hat{\lambda}\left(\frac{1}{e}\right) + \int\limits_{\lambda \geq \hat{\lambda}} e^{\phi\delta G} G'' \lambda f(\lambda) d\lambda + \phi \int\limits_{\lambda \geq \hat{\lambda}} e^{\phi\delta G}\left(c_e + \delta G'\lambda\right)G'f(\lambda)d\lambda \right\}.^{[73]}$$

Since $G', G'' > 0$, the first two terms: $G'(0)f\left(\hat{\lambda}\right)\hat{\lambda}\left(\dfrac{1}{e}\right)$ and $\int\limits_{\lambda \geq \hat{\lambda}} e^{\phi\delta G} G'' \lambda f_\lambda \, d\lambda$ are both positive.

The third term: $\phi \int\limits_{\lambda \geq \hat{\lambda}} e^{\phi\delta G}\left(c_e + \delta G'\lambda\right)G'f_\lambda \, d\lambda$ is positive if:

(i) $c_e + \delta G'\lambda > 0 \ \ \forall \ \lambda \geq \hat{\lambda}$; and then $\xi_{e\ell} > 0$ must hold.

(ii) If not, then there exists some $\tilde{\lambda} \geq \hat{\lambda}$ such that $c_e + \delta G'\lambda \geq 0 \ \forall \lambda \geq \tilde{\lambda}$, and $c_e + \delta G'\lambda < 0$ otherwise. Let $G'\left(\tilde{\lambda}\right) = K$, then:

$$\int\limits_{\lambda \geq \hat{\lambda}} e^{\phi\delta G}\left(c_e + \delta G'\lambda\right)G'f_\lambda \, d\lambda$$
$$= \int\limits_{\lambda \geq \hat{\lambda}} e^{\phi\delta G}\left(c_e + \delta G'\lambda\right)(G' - K)f_\lambda \, d\lambda + K \int\limits_{\lambda \geq \hat{\lambda}} e^{\phi\delta G}\left(c_e + \delta G'\lambda\right)G'f_\lambda \, d\lambda.$$

---

[73] See Appendix 2.

The first term is positive since $G' - K \geq 0$ when $\lambda \geq \tilde{\lambda}$, and $G' - K < 0$ otherwise, and thus the product of $(c_e + \delta G' \lambda)$ and $(G' - K)$ must always be positive. The second term is also positive as shown in footnote (52). Consequently we get $\int_{\lambda \geq \hat{\lambda}} e^{\phi \delta G} (c_e + \delta G' \lambda) G' f_\lambda \, d\lambda > 0$,

and thus $V_{e\ell} > 0$.

Q.E.D. ∎

Following from the **Corollary**, we can then show the next proposition.

Proposition 2.

A firm's production and emission both decrease as permit price rises, if it exhibits CARA, and its emission and permit holding are complements (i.e., $V_{e\ell} > 0$).

Proof.

From (2) and (5) we get:

$$\frac{dq}{de} = \frac{-c_{qe}}{c_{qq}} > 0$$

and

$$\frac{\partial q}{\partial p} = \frac{\partial q}{\partial e} \cdot \frac{\partial e}{\partial p}.$$

From (7) and adopting the functional form in (8), we can derive:

$$\frac{\partial e}{\partial p} = \frac{V_{e\ell} V_{\ell p}}{|H|} < 0$$

and

$$\frac{\partial q}{\partial p} = \frac{\partial q}{\partial e} \cdot \frac{\partial e}{\partial p} < 0.$$

Q.E.D. ∎

That is, when firms are CARA (as indicated in (8)), emission and production both

decrease as permit prices rises, namely, $\frac{\partial e}{\partial p}, \frac{\partial q}{\partial p} < 0$. The results are the same as in the case of

additive error -- as permit price rises, it is relatively less expensive to simply reduce its

pollution and production.

Next we exam the effect of the policy weight δ on a firm's production and pollution

decisions. First, we show that a firm's expected utility is decreasing as the government raises

the scale of penalty (while holding the expected fine unchanged).

Proposition 3.

The maximized utility for risk averse firms decreases as the policy weight δ increases.

Proof. (same as in the additive case, using Taylor's expansion.)

Recall equation (1a) (in terms of $q^* = q(e,t)$), and rewrite the optimized expected utility

in terms of $\left\{ \frac{\pi}{\delta}, \delta G \right\}$:

$$V(\delta) = u(R_0) + \frac{\pi}{\delta} \int_{\hat{\lambda}}^{\infty} \left[u(R_0 - \delta G) - u(R_0) \right] f(\lambda) d\lambda$$

Differentiate it w.r.t δ and invoke the envelope theorem, we get:

$$\frac{dV}{d\delta} = V_\delta = \frac{\pi}{\delta^2} \int_{\hat{\lambda}}^{\infty} \left[u(R_0) - u(R_0 - \delta G) - \delta G u'(R_0 - \delta G) \right] f(\lambda) d\lambda$$

Using Taylor's expansion, $u(R_0)$ can be expressed as:

$$u(R_0) = u(R_0 - \delta\tilde{G}) + \delta\tilde{G}u'(R_0 - \delta\tilde{G}) + \frac{u''(R)(\delta\tilde{G})^2}{2} \quad \forall \text{ some } R \in (R_0 - \delta\tilde{G}, R_0), \tilde{G} \equiv G(\lambda - \tilde{\lambda})$$

.

Substitute this into V_δ, and by risk aversion ($u'' < 0$), we get $\dfrac{dV}{d\delta} < 0$.

Q.E.D. ∎

To see the effect of increased penalty weight on firms' compliance decision, we use the permit-to-emission rate, $\hat\lambda$. Recall that given emission (and hence the production q, since $q^* = q(t, e)$), the firm's objective function can be rewritten in $\hat\lambda$:

$$V(\delta) = u(R_0) + \frac{\pi}{\delta}\int_{\hat\lambda}^{\infty}\left[u(R_1) - u(R_0)\right]f(\lambda)d\lambda ,$$

where $R_0 = tq^* - c(q^*, e) - pe\hat\lambda$, and $R_1 = R_0 - \delta G\big((\lambda - \hat\lambda)e\big)$.

The FOC w.r.t. $\hat\lambda$ is:

(10) $$V_{\hat\lambda} = -pe \cdot u'(R_0) + \frac{\pi}{\delta}\,pe\int_{\lambda \geq \hat\lambda}\left[\left(\frac{\delta G'}{p} - 1\right)u'(R_1) + u'(R_0)\right]f_\lambda\, d\lambda = 0$$

Proposition 4.

When firms exhibit CARA, the government can promote the degree of compliance (in the sense of permit to emission ratio, namely $\hat\lambda$), by increasing the weight on the penalty while holding the expected fine unchanged.

Proof.

Differentiate (10) w.r.t. δ and adopt CARA ($u'' = -\phi u'$):

(11)

$$V_{\hat\lambda\delta} = -\frac{\pi}{\delta^2}\,pe\int_{\lambda \geq \hat\lambda}\left[\left(\frac{\delta G'}{p} - 1\right)u'(R_1) + u'(R_0)\right]f_\lambda\, d\lambda$$

$$-\frac{\pi}{\delta}\,pe\int_{\lambda \geq \hat\lambda}\left(\frac{\delta G'}{p} - 1\right)u''(R_1)\cdot G\cdot f_\lambda\, d\lambda + \frac{\pi}{\delta}\,pe\int_{\lambda \geq \hat\lambda}\left(\frac{G'}{p}\right)u'(R_1)f_\lambda\, d\lambda$$

$$
= pe \left\{ \begin{array}{l} -\dfrac{\pi}{\delta^2} \displaystyle\int_{\lambda \geq \hat{\lambda}} \left(\dfrac{\delta G'}{p} - 1 \right) u'(R_1) f_\lambda \, d\lambda - \dfrac{\pi}{\delta^2} \displaystyle\int_{\lambda \geq \hat{\lambda}} u'(R_0) f_\lambda \, d\lambda \\[4mm] + \dfrac{\pi}{\delta^2} \displaystyle\int_{\lambda \geq \hat{\lambda}} \left(\dfrac{\delta G'}{p} \right) u'(R_1) f_\lambda \, d\lambda + \phi \dfrac{\pi}{\delta} \displaystyle\int_{\lambda \geq \hat{\lambda}} \left(\dfrac{\delta G'}{p} - 1 \right) u'(R_1) \cdot G \cdot f_\lambda \, d\lambda \end{array} \right\}
$$

$$
= pe \left\{ \begin{array}{l} -\dfrac{\pi}{\delta^2} \displaystyle\int_{\lambda \geq \hat{\lambda}} \left(\dfrac{\delta G'}{p} \right) u'(R_1) f_\lambda \, d\lambda + \dfrac{\pi}{\delta^2} \displaystyle\int_{\lambda \geq \hat{\lambda}} \left(\dfrac{\delta G'}{p} \right) u'(R_1) f_\lambda \, d\lambda \\[4mm] + \dfrac{\pi}{\delta^2} \displaystyle\int_{\lambda \geq \hat{\lambda}} \left[u'(R_1) - u'(R_0) \right] f_\lambda \, d\lambda + \phi \dfrac{\pi}{\delta} \displaystyle\int_{\lambda \geq \hat{\lambda}} \left(\dfrac{\delta G'}{p} - 1 \right) u'(R_1) \cdot G \cdot f_\lambda \, d\lambda \end{array} \right\}
$$

$$
= pe \left\{ \dfrac{\pi}{\delta^2} \displaystyle\int_{\lambda \geq \hat{\lambda}} \left[u'(R_1) - u'(R_0) \right] f_\lambda \, d\lambda + \phi \dfrac{\pi}{\delta} \displaystyle\int_{\lambda \geq \hat{\lambda}} \left(\dfrac{\delta G'}{p} - 1 \right) u'(R_1) \cdot G \cdot f_\lambda \, d\lambda \right\}
$$

The first term in the bracket is positive by risk aversion.

Using the same technique in the Corollary's proof, we can prove the second term is positive as well.

That is, $V_{\hat{\lambda} \delta} \big|_{V_{\hat{\lambda}} = 0} > 0$: as the government raises its penalty weight, firms' compliance, in

the sense of permit to pollution rate, increases.

$$Q.E.D. \blacksquare$$

Whether the rise in compliance (in response to the rise of policy weight) comes from the drop in pollution or the increase in permit holding, nonetheless, is not clear, since the effects of increased policy weight on the firms' optimal decisions is equivocal. Proposition 4 notes this particular point.

Proposition 5.

   When firms exhibit CARA with the functional form in (8), changes of the policy weight, $\delta \geq \pi$, have ambiguous impacts on the firms' optimal choices.

Proof.

Using the technique in Proof 1, we can prove that $V_{\ell\delta} > 0$ (same as in the additive error case) and $V_{e\delta} < 0$.

(i) From (4a):

$$V_{\ell\delta} = \frac{p\pi}{\delta^2} \int_{\hat{\lambda}}^{\infty} \left(u'(R_1) - u'(R_0) \right) f_\lambda \, d\lambda + \frac{p\pi}{\delta} \int_{\hat{\lambda}}^{\infty} u''(R_1) G \cdot f_\lambda \, d\lambda - \pi \int_{\hat{\lambda}}^{\infty} u''(R_1) G \cdot G' f_\lambda \, d\lambda$$

Adopt CARA ($u'' = -\phi u'$) and simplify:

$$V_{\ell\delta} = \frac{p\pi}{\delta^2} \int_{\hat{\lambda}}^{\infty} \left(u'(R_1) - u'(R_0) \right) f_\lambda \, d\lambda - \pi \left[\int_{\hat{\lambda}}^{\infty} u''(R_1) G \left(G' - \frac{p}{\delta} \right) f_\lambda \, d\lambda \right]$$

$$= \frac{p\pi}{\delta^2} \int_{\hat{\lambda}}^{\infty} \left(u'(R_1) - u'(R_0) \right) f_\lambda \, d\lambda + \phi\pi \left[\int_{\hat{\lambda}}^{\infty} u'(R_1) G \left(G' - \frac{p}{\delta} \right) f_\lambda \, d\lambda \right].$$

The first term is always positive by risk aversion.

(a) If $G' > \dfrac{p}{\delta}$ $\forall \lambda > \hat{\lambda}$, then the second term is everywhere positive.

(b) Otherwise, there exists some $\tilde{\lambda}$ such that $G' \gtreqqless \dfrac{p}{\delta}$ $\forall \lambda \gtreqqless \tilde{\lambda}$, since $G'' > 0$. Let $\tilde{G} = G(\tilde{\lambda})$, then:

$$\int_{\hat{\lambda}}^{\infty} u'(R_1) G \left(G' - \frac{p}{\delta} \right) f(\lambda) d\lambda$$

$$= \int_{\hat{\lambda}}^{\infty} u'(R_1)(G - \tilde{G}) \left(G' - \frac{p}{\delta} \right) f(\lambda) d\lambda + \tilde{G} \int_{\hat{\lambda}}^{\infty} u'(R_1) \left(G' - \frac{p}{\delta} \right) f(\lambda) d\lambda$$

where the first term is positive since $G \gtreqqless \tilde{G}$ whenever $G' \gtreqqless \dfrac{p}{\delta}$, and the second term is positive by (4):

$$\int_{\hat{\lambda}}^{\infty} u'(R_1) \left(G' - \frac{p}{\delta} \right) f(\lambda) d\lambda = \left(1 - \frac{\pi}{\delta} + \frac{\pi}{\delta} F(\hat{\lambda}) \right) \frac{pu'(R_0)}{\pi} > 0$$

$$\Rightarrow V_{\ell\delta} > 0.$$

(ii) $V_{e\delta} < 0$ can be proved using the same technique.

(iii) From the FOCs:

$$\frac{\partial e}{\partial \delta} = \frac{-V_{e\delta}V_{\ell\ell} + V_{e\ell}V_{\ell\delta}}{|H|}$$

$$\frac{\partial \ell}{\partial \delta} = \frac{-V_{ee}V_{\ell\delta} + V_{e\ell}V_{e\delta}}{|H|}$$

and

$$\frac{\partial q}{\partial \delta} = \frac{\partial q}{\partial e} \cdot \frac{\partial e}{\partial \delta}.$$

Yet by the corollary, $V_{e\ell} > 0$, and the signs of $\left\{ \frac{\partial e}{\partial \delta}, \frac{\partial \ell}{\partial \delta}, \frac{\partial q}{\partial \delta} \right\}$ cannot be determined.

Q.E.D. ∎

When the government lowers monitoring efforts and raises penalty, changes in firms' pollution, production and permit demand depend on their preferences: on one hand, in this particular case, because $V_{e\ell} > 0$, i.e., permit and pollution move in the same direction at optimum, it is not clear about how the government's policy will impact on firms' pollution and permit holding decisions; on the other, if given different functional form such that permit and pollutions move in the opposite direction (i.e., $V_{e\ell} < 0$), one can conclude that firms' emission and production decrease, while the permit demand increases as the government raises the penalty scale. Yet from **Proposition 4**, we see that the government can still promote compliance by raising the penalty scale, though the major attribution of this compliance increase is ambiguous. This result can be extended to the effect of audit frequency change as well.

Note that, if there is an increase in permit price (which may be due to more stringent regulation that results in decreased permit supply), it will lead to decreases in emission, production and permit demand, for as the permit equilibrium price rises, it becomes relatively less expensive to simply produce/pollute less.[74]

Next, we address the impacts of government's regulatory instruments on a firm's optimal decisions in the following chapter, and in Chapter 5 we present the numerical simulation results of the model.

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[74] Keep in mind that, though the model assumes the disturbances come from measurement error, the source of uncertainty has no impact on our results. That is, our analysis is robust if the firms' (observed) emission is subject to any stochastic factors (e.g., production shocks, weather impacts, etc.)

CHAPTER 5: POLICY AND EQUILIBRIUM

Having spent the previous chapters on individual firms' behavior, we next set forth to explore the government's best policy choices and the industry equilibrium.

Because the measurement error and the lack of risk markets, the government uses the permit market to mitigate both the environmental damage and the risk. Hence in the long-run when there is free-entry/exit, while raising the violation penalty may alleviate the pollution, it may as well deter entries (and/or encourage exits) of the industry, reciprocally changing the equilibrium and creating inefficiency in the economy. To see the effects of endogenous industry size, we assumed that firms are of different efficiency levels, which reflect in their cost functions (as in the very beginning of the model). Explicitly, let firm i's cost function be indexed by it's type $\theta_i \in \left(\underline{\theta}, \overline{\theta} \right)$:

(12) $\qquad c_i(q,e) = \theta_i c(q,e),$

while arranging firms according to their types such that $c_1 \leq c_2 \leq \dots$, and hence $\theta_1 \leq \theta_2 \leq \dots$, i.e., the marginal firm in the industry would be the least efficient firm.

For the government to assess its best policy instruments, first we need to describe the equilibrium in the economy. Suppose two goods, J and q, are produced in the economy, where J is produced at constant labor cost. Individuals in the economy can choose to become an entrepreneur, founding a firm to produce q by means of labor and pollution, or to devote his labor to produce J. For illustration purpose, we use a simple numerical example as the following:

Let the wage rate be w. The production function for q is $q = \left(\dfrac{e^{2/3} x}{\theta} \right)^{1/2}$ (for decreasing

return to scale), where x is the labor input. Then the cost function is: $c_i(q, e) = w\theta_i \dfrac{q^2}{e^{2/3}}$,

where θ_i can is the efficiency parameter. Profits from making q is as defined before:

$$R_0 = tq - w\theta_i \frac{q^2}{e^{2/3}} - p\ell \text{ , and}$$

$$R_1 = R_0 - \delta G\left((\lambda - \hat{\lambda})e \right) \text{ if fined.}[75]$$

The equilibrium condition for a risk averse firm (individual) is such that his expected

utility from setting up a firm is equal to the expected utility from earning w. In addition, let

the utility function be as defined in (8), then:

(13) $\qquad \max\limits_{\ell, e, q} E(u) = E\left(u(w) \right) = -\dfrac{\exp(-\phi w)}{\phi}$.

Equation (13) determines the marginal firm's type-$\hat{\theta}$. To describe the social welfare

function, assume the economy consumes only the nominal goods J and exporting all products

produced by entrepreneurs. Individuals earning wage w spent it all on the nominal goods J.

So the ordinal utility is $w + T - \alpha E^2$, where $E = N \displaystyle\int_{\underline{\theta}}^{\hat{\theta}} e^*(\theta) h(\theta) d\theta$ is the aggregate emission

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[75] From (5): $q^*(e, t) = \dfrac{t e^{2/3}}{2w\theta}$, and $R_0 = \dfrac{t^2 e^{2/3}}{4w\theta} - p\ell$.

($h(\cdot)$ the density function of θ, and N is the number of total individuals in the economy), α is the damage fraction perceived by individual, and T is the transfer of fines paid by firms that individuals received from the government. Let $V^*(\theta)$ be the maximized expected utility for firm of type-θ at equilibrium,. Then the social welfare function is:

(15)

$$\max_{\{\pi,\delta,L\}} SW = N\left(1-H(\hat{\theta})\right)u(w+T-\alpha E^2)+N\int_{\underline{\theta}}^{\hat{\theta}} \underset{\lambda}{E}\left[u\left(R(\theta)\right)\right]h(\theta)d\theta$$

$$= \exp\left(-\phi(T-\alpha E^2)\right)\left\{-N\left(1-H(\hat{\theta})\right)\frac{\exp(-\phi w)}{\phi}+N\int_{\underline{\theta}}^{\hat{\theta}} V^*(\theta)\cdot h(\theta)d\theta\right\},$$

where $V^*(\theta) = \max_{e,\ell,q} \underset{\lambda}{E}\left(u(R)\right)$, and the second equality follows from the utility functional form in (8).

Properties of $V^*(\theta)$ can be derived from previous propositions. First, from **Proposition 1** in Chapter 3, the firm's permit demand decreases as permit price increases, when the firm exhibits CARA, i.e., $\dfrac{\partial \ell}{\partial p} < 0$.

From the permit market clearing condition: $N\int_{\underline{\theta}}^{\hat{\theta}} \ell(\theta)\cdot dH(\theta) = L$, we get:

(16)
$$\frac{\partial p}{\partial L} = \frac{1}{N\int_{\underline{\theta}}^{\hat{\theta}} \dfrac{\partial \ell(\theta)}{\partial p}\cdot dH(\theta)} < 0$$

Now as assumed, due to free trade, the product market is open such that the product price

is exogenous, then: $\dfrac{\partial V^*(\theta)}{\partial p} = -\ell^*(\theta)z^*(\theta) < 0$,[76] and hence we get:

(17) $$\frac{\partial V^*(\theta)}{\partial L} = \frac{\partial V^*(\theta)}{\partial p} \cdot \frac{\partial p}{\partial L} > 0 .$$

That is, if the government increases the number of total permits, resulting in lower

equilibrium permit price, then given the efficiency level θ , the expected utility $V^*(\theta)$ rises,

which entices entry of less efficient firms into the industry. Aggregate production increases

due to industry expansion till the equilibrium is restored (as firms entering, permit price

eventually goes up).

Moreover, by Envelope Theorem:

(18) $$\frac{\partial V^*(\theta)}{\partial \pi} = \frac{1}{\delta} \int_{\hat{\lambda}}^{\infty} \left[u(R_0 - \delta G) - u(R_0) \right] f_\lambda \, d\lambda < 0 .$$

And from **Proposition 3** in Chapter 3:

(19) $$\frac{\partial V^*(\theta)}{\partial \delta} = -\frac{\pi}{\delta^2} \int_{\hat{\lambda}}^{\infty} \left[u(R_0 - \delta G) - u(R_0) \right] f_\lambda \, d\lambda - \frac{\pi}{\delta} \int_{\hat{\lambda}}^{\infty} u'(R_0 - \delta G) G \cdot f_\lambda \, d\lambda < 0 .$$

Then, if allow the industry size to be endogenous, when the government raises its audit

frequency or/and the penalty weight, the marginal firm's expected utility shrinks and thus

may leave the industry, squeezing the industry size further smaller.

The marginal effect of policy weight on the aggregate production and the pollution is

ambiguous. As noted in **Proposition 5** in Chapter 3, the answer to whether an individual

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[76] See Appendix 3.

firm's output and emission decrease or increase along with the increased policy weight δ, is not a clear-cut one. Unlike the additive error case, where rising δ squeezes the industry size, while suppressing the aggregate production and pollution fewer firms all the same time, we cannot determine whether it will have the same impact on aggregate output and pollution in the multiplicative error case, even as the industry dwindles.

The ambiguity leads us to the next chapter, where we use simulations to demonstrate the effects of government's policy choices on the industry and moreover, to determine the possible optimal policy in terms of number of permits and the policy weight.

CHAPTER 6: NUMERICAL SIMULATIONS

Since an analytic solution is unattainable in the multiplicative case, we use numerical approximation to get a clearer understanding on the behavior of the choice variables: emission, permit, and production.

As in previous chapters, assume a CARA utility function:

$$u(R) = -\frac{\exp(-\phi R)}{\phi}, \ \phi > 0.$$

Following from the last chapter, assume the production function is a function of labor input and pollution:

$$q_i = \left(\frac{e^{2/3} x}{\theta_i}\right)^{1/2}.$$

Then the cost function is:

$$c_i(q, e) = w\theta_i \frac{q^2}{e^{2/3}},$$

where $\theta_i \in \left(\underline{\theta}, \overline{\theta}\right)$ is firm-i's type (in terms of production efficiency).

The entry condition is such that the expected utility for an entrepreneur is greater or equal to the expected utility of him being employed and earning wage w:

$$\max_{\ell, e, q} E(u) \geq E\big(u(w)\big) = -\frac{\exp(-\phi w)}{\phi}.\text{[77]}$$

Firms chooses production q, emission e and permit holding ℓ, subject to the entry condition above.

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[77] This is true if we assume iid for λ: by iid, the government's transfers and the pollution damage can be treated as independent of λ, so the entrepreneurs only care about the realized utility resulted from production.

Assume a quadratic penalty function:

$$G(v) = 5v^2 + 10v,$$

where $v = \lambda e - \ell$ is the measured violation.

Social welfare is the sum of utility from labor workers (who earn the constant wage w) and the expected utility from entrepreneurs:

$$\max_{\{\pi,\delta,L\}} SW$$

$$= -\frac{N}{\phi} \left\{ \int_{\hat{\theta}}^{\bar{\theta}} \exp\left[-\phi\left(w + \frac{T}{N} - \alpha E^2\right)\right] \cdot h(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} \exp\left[-\phi\left(v^*(\theta) + \frac{T}{N} - \alpha E^2\right)\right] \cdot h(\theta)d\theta \right\}, {}^{78}$$

$$= \left(-\frac{N}{\phi}\right) \exp\left(-\phi(\frac{T}{N} - \alpha E^2)\right) \left\{ \left(1 - H(\hat{\theta})\right) \exp(-\phi w) + \int_{\underline{\theta}}^{\hat{\theta}} V^*(\theta) \cdot h(\theta)d\theta \right\}$$

where $V^*(\theta) = \max_{e,\ell,q} E(R)$ is the entrepreneur's optimal expected profit, and T is the lump-sum transfer of fine and permit revenue collected.[79]

Parameters

Because firms are price takers (followed from the last chapter, it is a small open economy), let product price $t = 10$ and the wage rate $w = 1$.[80] Also assume the absolute risk

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[78] The second equality follows as λ is iid.

[79] Through out the numerical examples we also assume the population's type is uniformly distributed over the interval $[1,6]$, i.e., $h(\theta) \sim U[1,6]$. For simplicity, we assume the population size $N = 5$. Note that since pollution is a pure public good, social welfare is also a function of the size of population,

[80] As laid out in Chapter 4, there are two goods produced in the economy, one with pollution and the other is a clean good. The clean (numeraire) good is produced at constant marginal product of labor. The (dirty) good price is chosen to ensure the model is well-defined, and the permit demand is positive.

aversion coefficient $\phi = 1$ for simplicity, and the audit probability $\pi = 0.5$.[81] The benchmark

parameterization is summarized in the table below:

Table 1. Initial Parameterization					
t	w	π	ϕ	δ	α
10	1	0.5	1	1	0.1

Based on the parameters, we proceed to calculate the optimal amount of permits for the

economy, and then investigate the impacts of parameters on firm-level choices, and the

industry evolution around the optimum.

1. Methodology

First, we calculate the (welfare-maximizing) solutions for the model with no

measurement error (and thus no real uncertainty) as the baseline, and then proceed to assess

the impact of uncertainty.

In Mathematica, we began by establishing a module to compute an individual firm's

choices dependent on the parameters of interest, i.e. the equilibrium permit price, the type of

the firm, the audit probability and the policy weight. Given the results we then computed the

aggregate permit demand dependent on the marginal type of firm, which was determined by

the free-entry condition. Next, by equating the permit demand to the permit supply we got the

equilibrium permit price, from which the individual and aggregate choices could be

determined. Social welfare is calculated subsequently. With loop-like iterations we

determined the welfare-maximizing permit supply and the corresponding values of variables,

which formed the starting point of the comparative statics.

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[81] The values are chosen to make the simulations easier to manipulate.

2. Benchmark Scenario: Certainty

In the deterministic world, all firms are compliant and the government maximizes social welfare via adjusting the number of permits. A firm's pollution and production choices are functions of permit price, product price and its type: $e^*(\theta) = \dfrac{t^6}{216 p^3 \theta^3 w^3}$, and

$q^*(\theta) = \dfrac{t^5}{72 p^2 w^3 \theta^3}$. A firm's maximized profit is then defined as:

$$v^*(\theta) = t \cdot q^*(\theta) - c\left(q^*(\theta), e^*(\theta)\right) - p \cdot e^*(\theta)$$

Adopting the CARA functional form, the social welfare is:

$$SW = \frac{-N}{\phi} \cdot \exp\left[-\phi\left(\frac{T}{N} - \alpha E^2\right)\right]\left\{\int_{\hat{\theta}}^{\bar{\theta}} \exp(-\phi w) h(\theta) d\theta + \int_{\underline{\theta}}^{\hat{\theta}} \exp\left(v^*(\theta)\right) h(\theta) d\theta\right\},$$

where $E = N\int_{\underline{\theta}}^{\hat{\theta}} e(\theta) h(\theta) d\theta$ is the aggregate pollution, with $\hat{\theta} = \dfrac{t^2}{6 \cdot 2^{\frac{1}{3}} p^{\frac{2}{3}} w^{\frac{1}{3}}}$ (determined by the

free entry condition: $v(\hat{\theta}) = w$).

Equating the aggregate pollution to the permit supply L , we can then express the equilibrium permit price in terms of L and maximize SW with respect to it.

Given the parameterizations, we simulate the welfare-maximized permit supply, equilibrium permit price and aggregate production through simulations.

Figure 1.

Social Welfare

Figure 2.

Permit Price

Figure 3.

Production

output

Figure 4.

Marginal θ

θhat
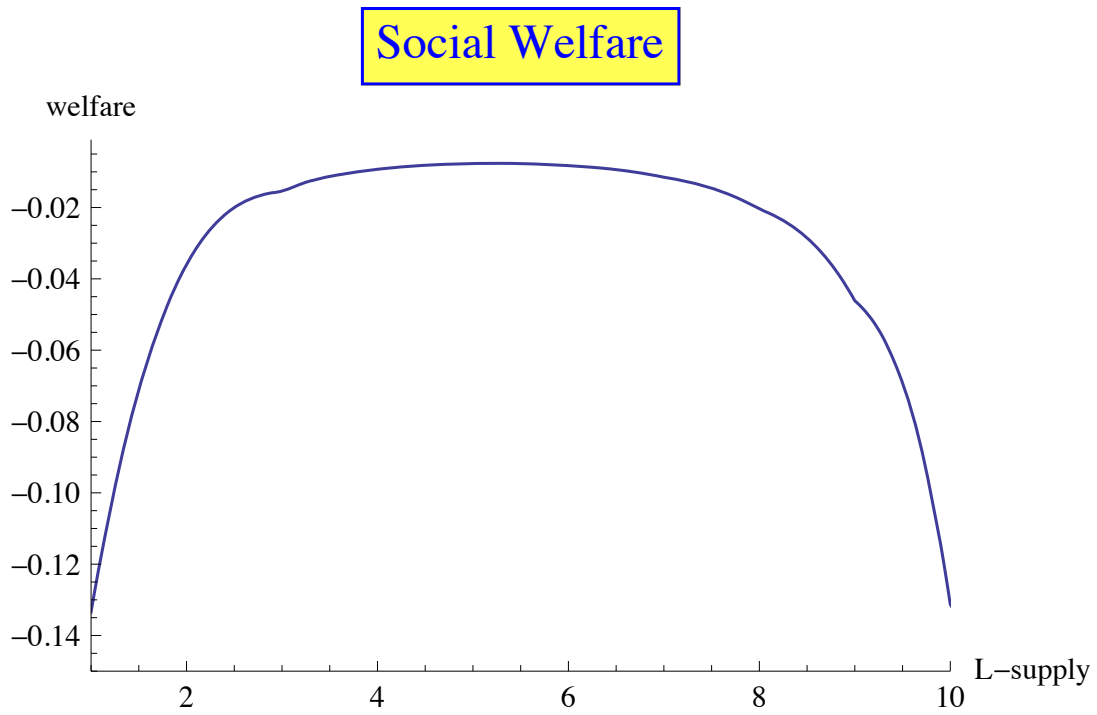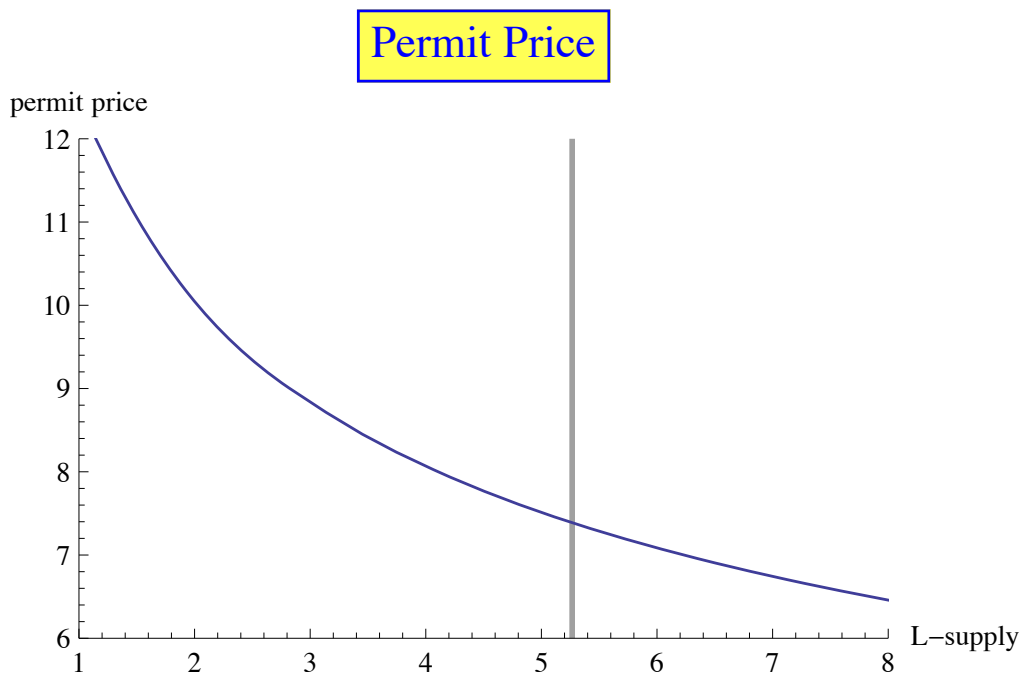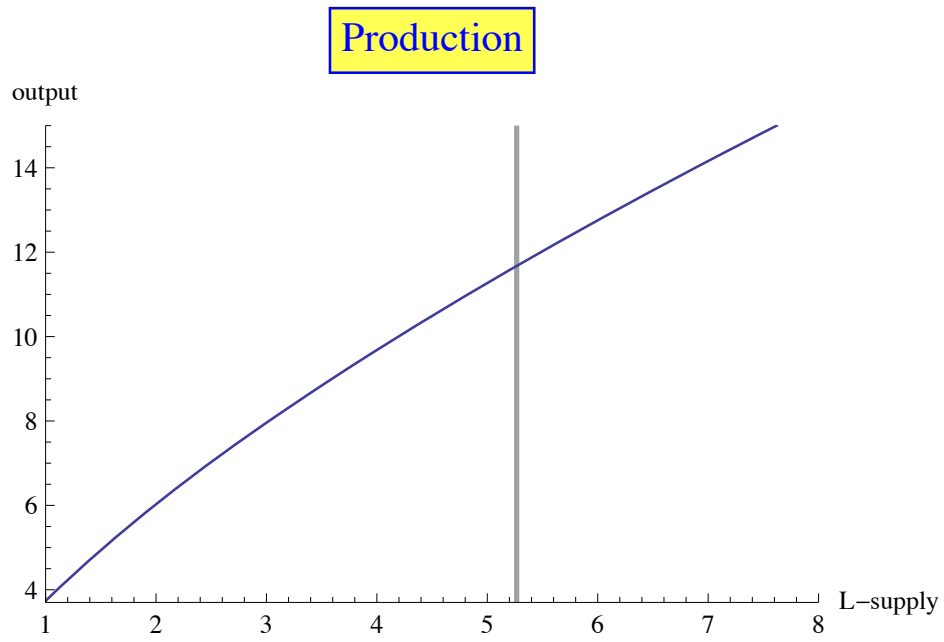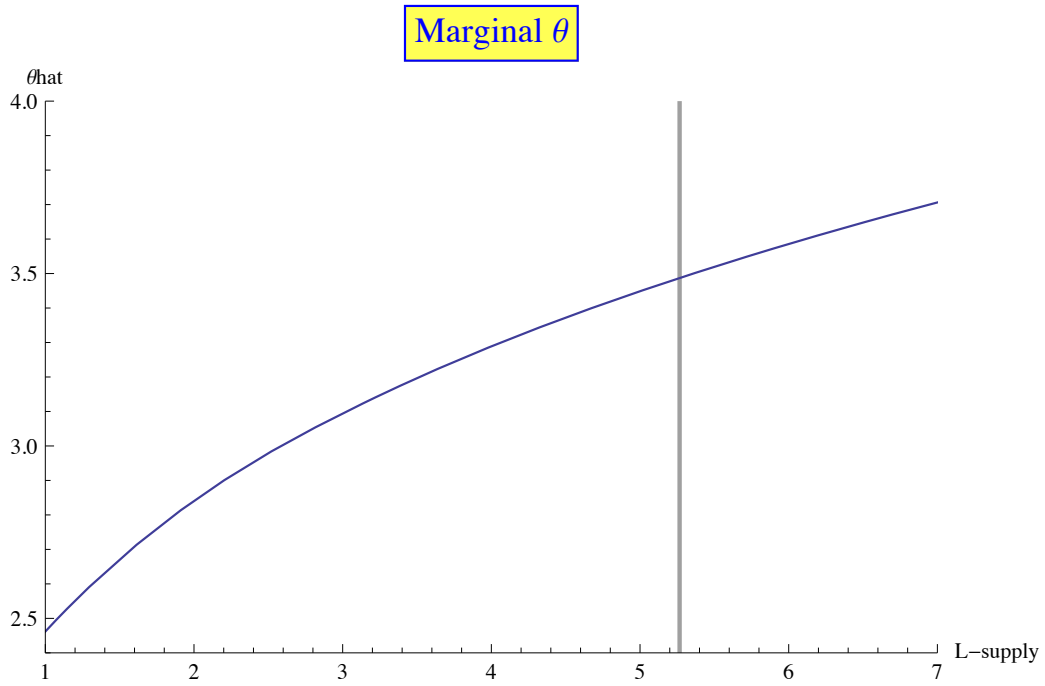
The optimal output, pollution and marginal type of the industry is summarized below:

Table 2. Optimal Variable Values under Certainty				
permit supply	permit price	production	marginal $\hat{\theta}$	welfare
5.266	7.389	11.674	3.487	-0.0076

Keep in mind that when there is no real uncertainty (such as the measurement error), a government's best policy is to maximize the penalty, while minimizing the audit probability (provided that there is no cost to fining, but monitoring is costly). The marginal firm's type is increasing as permit supply goes up.

3. Optimal Number of Permits

3.1 Industry Equilibrium

Next we introduce uncertainty, i.e., adding the measurement error λ into the model. For simplicity, we assume the error term λ follows the uniform distribution: $\lambda \overset{iid}{\sim} U(0,2)$.[82]

Given the parameters defined in Table 1, we simulate the permit demand function (which is still downward sloping in the permit price), and find the number of permits that maximize the social welfare function, as in the certainty case. Naturally, the equilibrium permit price decreases as the government increases the total number of permits, Social welfare approaches a local maximum around $L \approx 9$, and then begins decreasing afterwards.[83] That is, if the

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[82] Note that λ cannot be negative, since negative "observed pollution" is meaningless in the context of our model.

[83] When there is only a little uncertainty (e.g. $\lambda \sim U[0.95, 1.05]$), by allowing the audit probability to be large enough (e.g. $\pi = 0.99$), the optimal variable values are close to those of the certainty case.

initial permit supply is low (e.g., starting from the point when $L=1$), increasing the number of permits can raise social welfare through boosted production, despite increased pollution.

The optimal values of the aggregate variables are summarized in **Table 3**:

Table 3. Optimal Variable Values under Error					
permit supply	permit price	emission	production	marginal $\hat{\theta}$	welfare
9.432	3.821	5.134	11.550	3.921	-0.0102

The following graphs depict the optimal number of permits and the corresponding industry equilibrium (when the policy set of audit frequency is set to be 0.5 and the policy weight $\delta=1$). The thick gray line marks the optimal permit supply.

Figure 5.
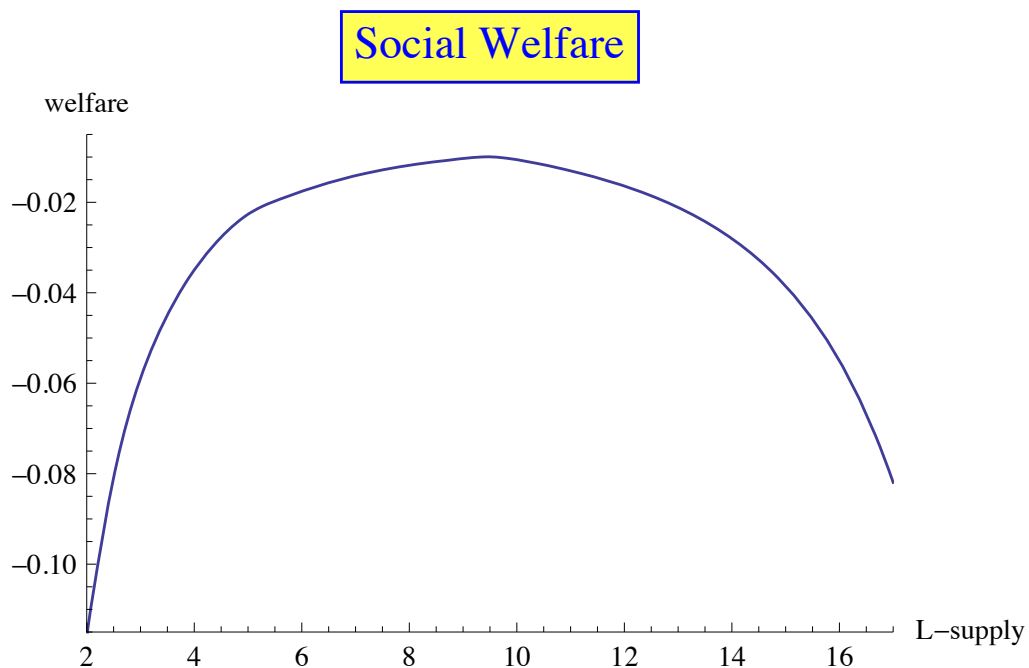
Figure 6.
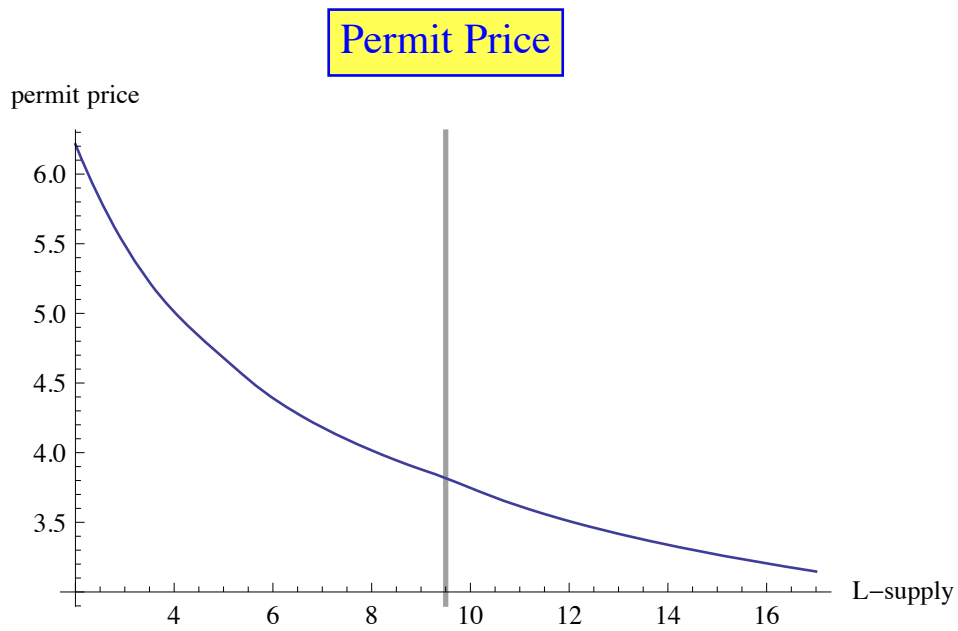
Permit Price

The equilibrium permit price and optimal industry size are determined by equating permit demand and supply, and the free entry condition respectively.

Figure 7.

Cutoff θ
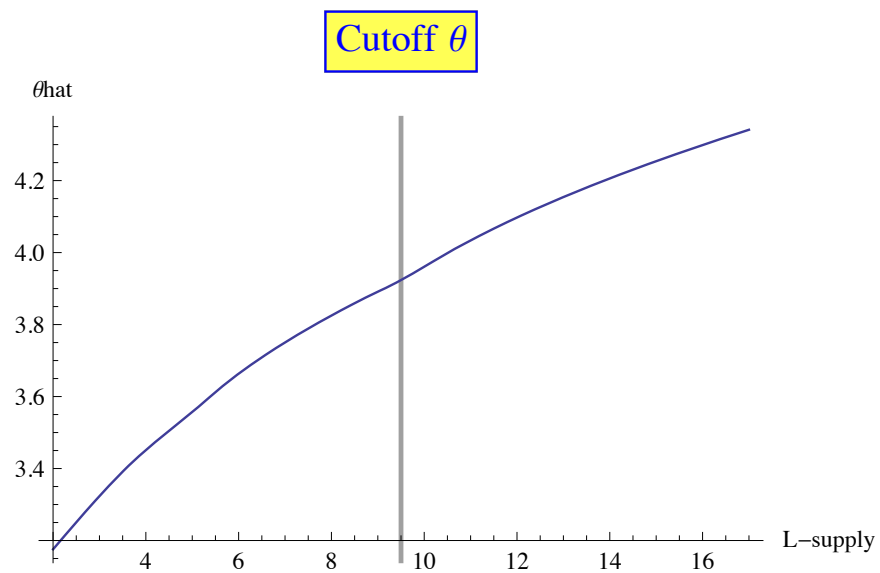
In contrast to the deterministic case, the social welfare ranking is lower when there is real uncertainty (resulting from measurement error). As the analytical model indicated, the industry with risk averse firms will pollute less than the intended amount (number of permits issued, that is), implying firms are hedging against the possibility of being wrongfully fined via permits.

As the number of permits increases, the total production and emissions will go up, and the industry expands as less efficient firms enter the market (indicated by the increased marginal $\hat{\theta}$). See the graphs below.

Figure 8.

Aggregate Production

Figure 9.

Aggregate Emission
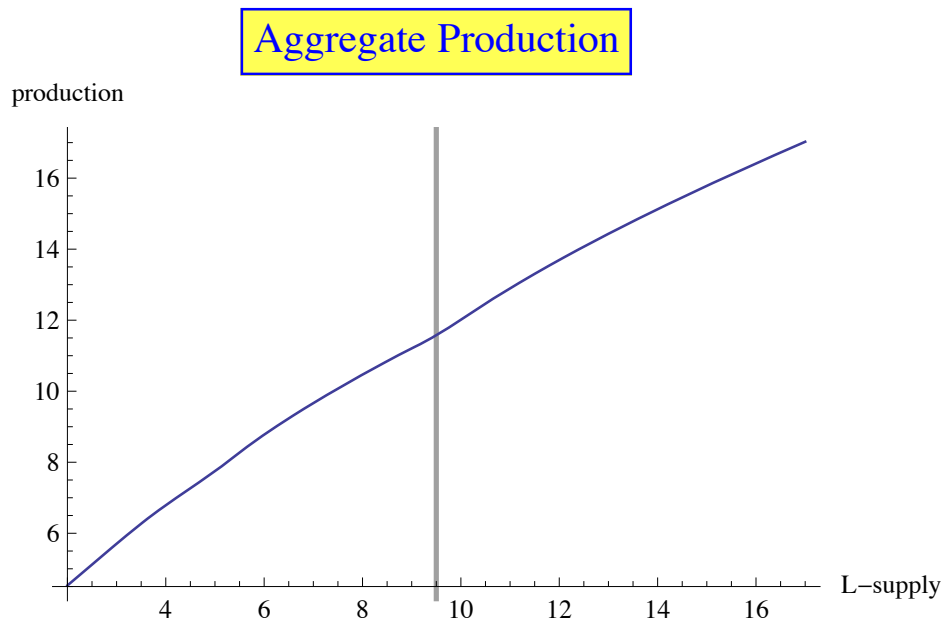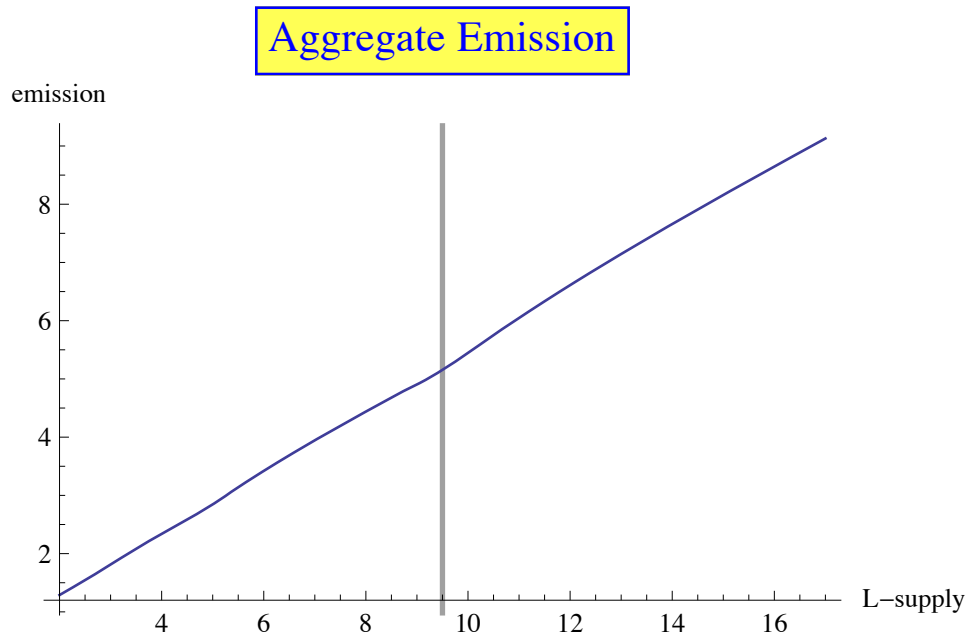
3.2 Individual Firms' Optimal Choices

With the equilibrium price and industry size, we inspect firms' choices at equilibrium against their types. (The thick gray lines in the graphs mark the marginal type $\hat{\theta}$.)

- *Production q and Pollution e*

Simulations confirms that a less efficient firm (i.e. whose type value θ is larger) produces and pollutes less.

- *Permit Demanded ℓ and Permit-to-pollution Ratio, $\hat{\lambda}$*

A firm tends to hold more permits if it is more efficient. The permit-to-pollution ratio, $\hat{\lambda}$, is also larger for more efficient firms, for firms produce and pollute more face higher risk of being fined, and hence they tend to maintain higher level of compliance.

Figure 10.
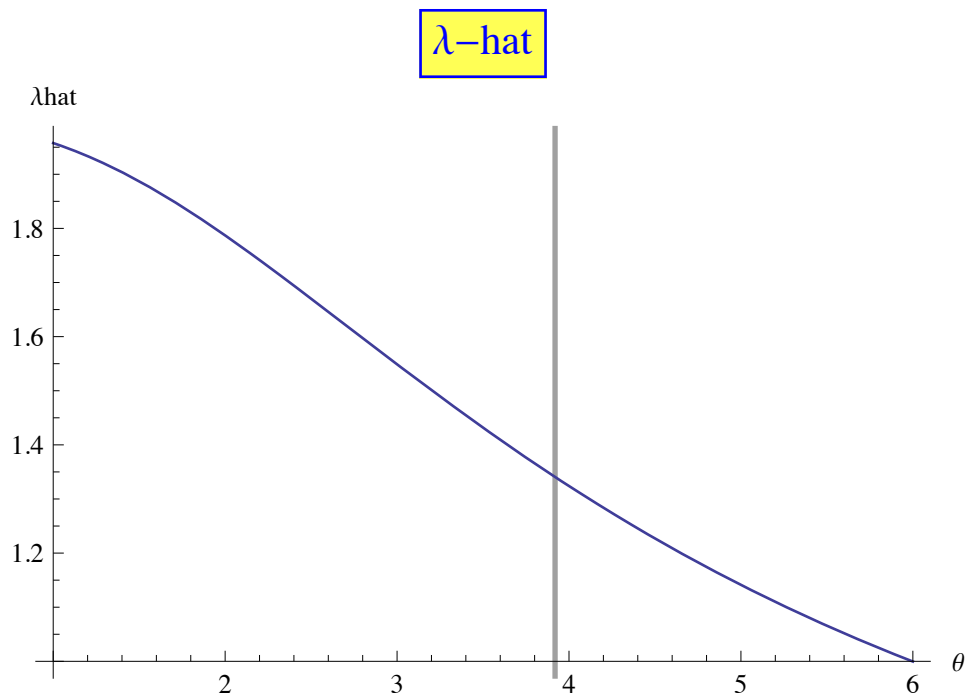
From here we simulate the impacts of changes in the parameters to the industry: the policy weight, and the audit frequency alone. Throughout this chapter, we use the firm of type-$\theta = 3$ as the sample to illustrate an individual firm's responses to these changes at equilibrium (graphs on the left-hand side) in contrast to the aggregates (graphs on the right-hand side).

4. Policy Impacts: Comparative Statics [84]

4.1 Policy Weight, δ [85]

In *Proposition 5* of **Chapter 3** we concluded that the impact of the policy weight on firms' production and pollution decisions is ambiguous. Here we use the specific numerical example in the hope of addressing that ambiguity and the resulting industry equilibrium.

- *Emission and Production: an individual firm*

Individual emission and production both fall as the weight on the penalty increases. That is to say, when the government raises the penalty, and decreases the audit frequency at the same time (while holding the expected fine unchanged), firms react by cutting back their emission and the resulting production, implying a risk-averse firm is more sensitive to the scale of punishment, which is consistent with the behavior of firms in the additive error case.

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[84] In the simulations for this section, we fixed the number of permit at its socially optimal value given the values of parameters specified in **Table 1**, and make changes only to the policy weight or the audit frequency.

[85] Here the policy weight δ is designed in the sense that the government raises the weight on penalty, but decreases the audit frequency at the same rate. That is, the policy set of audit frequency and the penalty is $\left\{\frac{\pi}{\delta}, \delta G\right\}$, such that the government can change the weight of those two instruments, while holding the expected fine unchanged.

Figure 11.

Emission, $\theta = 3$
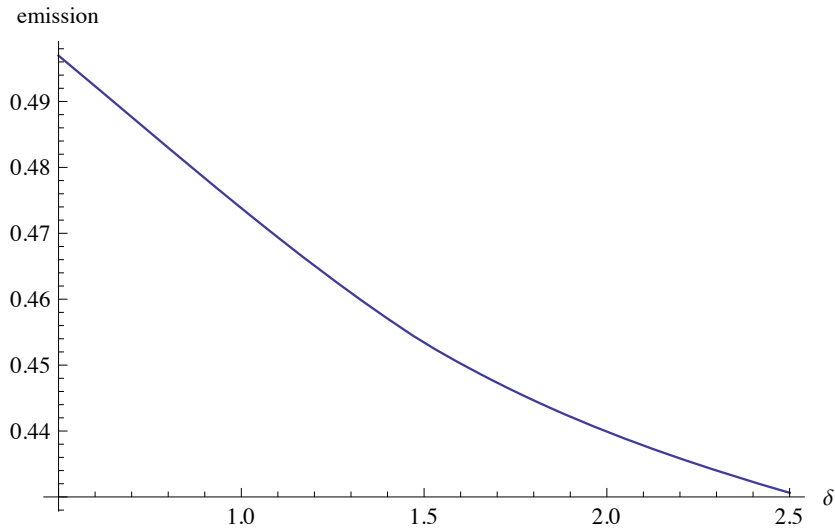
emission

Figure 12.

Production, $\theta = 3$
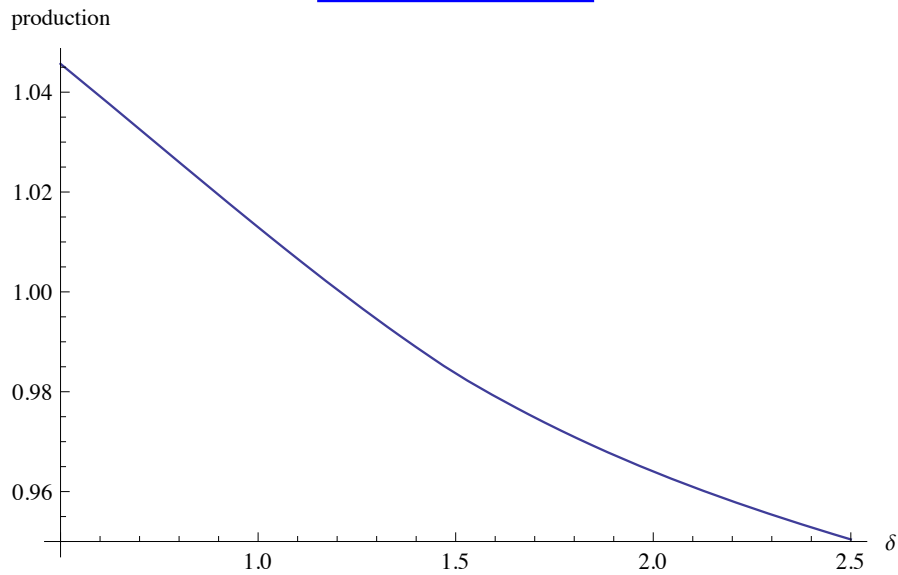
production

- *Permit Demanded and Permit Price*

As the weight on the penalty increases, the individual firm's permit demanded falls. The

trend corresponds to that of the declining emission/production: since a firm is cutting back its

emission/production in response to the rising policy weight, it no longer needs to hold as many permits to maintain its compliance level. Consistently, the equilibrium permit price also falls (provided the number of total permits is unchanged).

Figure 13.

Permit Demanded, $\theta = 3$

Figure 14.

Permit Price
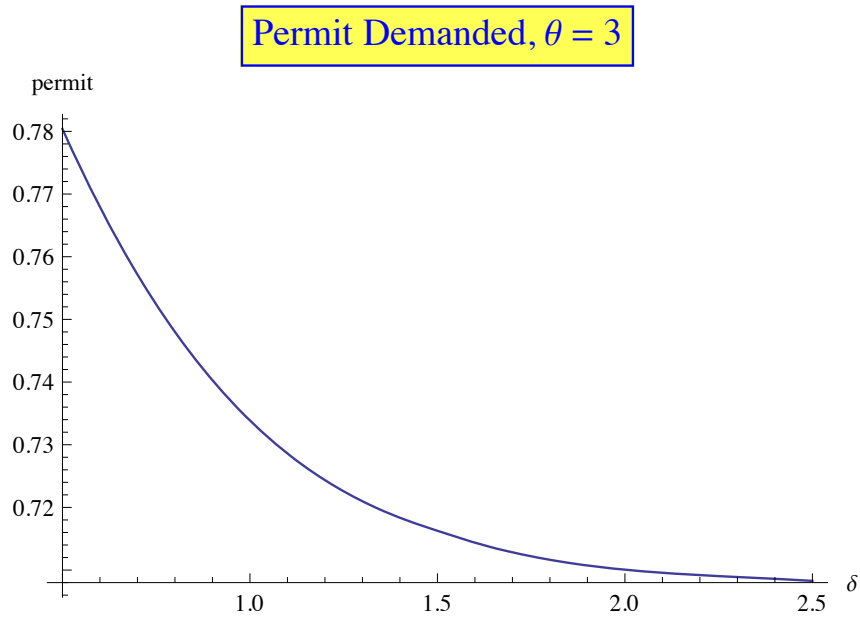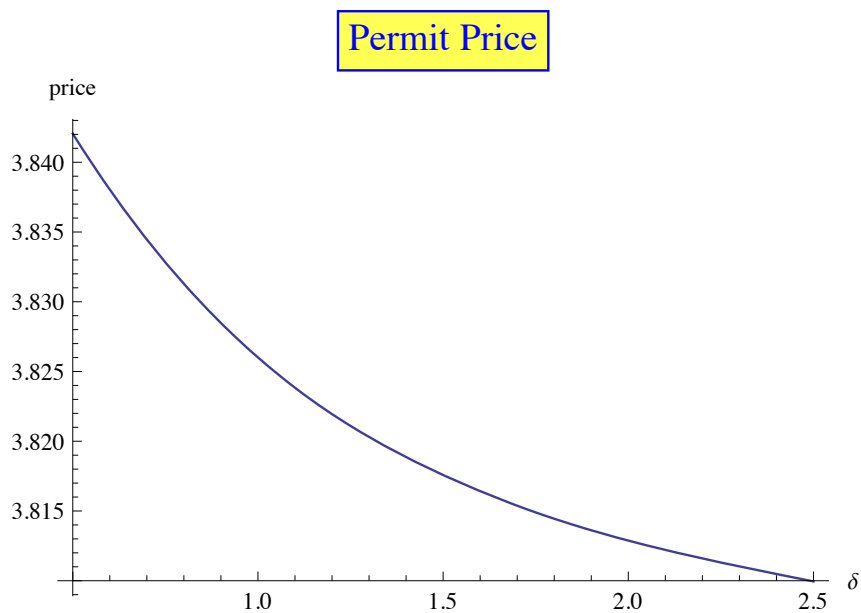
• *Permit-to-pollution ratio* $\hat{\lambda}$

Though firms cut their permit holding in the course of rising policy weight, the degree of

compliance in terms of permit-to-pollution ratio is still increasing in the severity of penalty.

This is consistent with the claim in *Proposition 4* in **Chapter 3**.

Figure 15.

$\lambda-\text{hat}, \theta = 3$
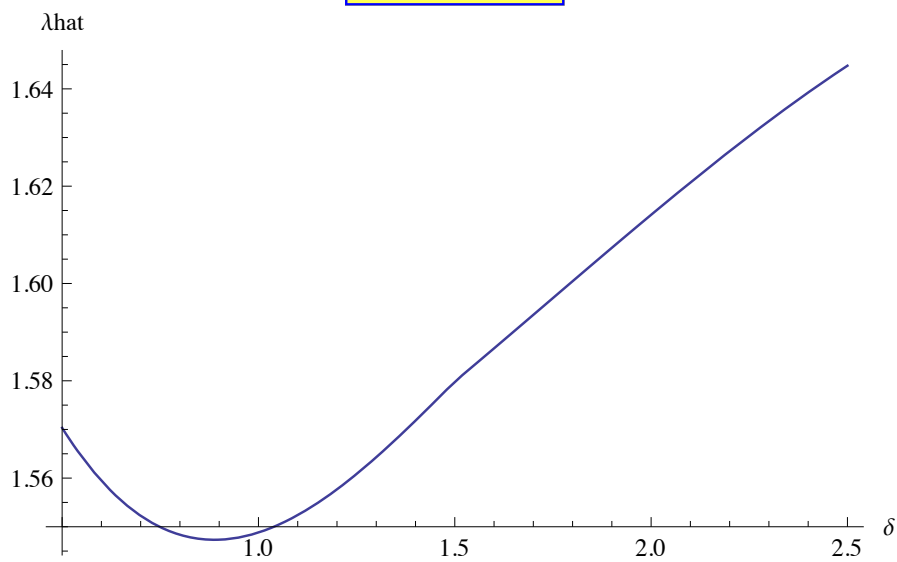
• *Marginal Type* $\hat{\theta}$

The industry's marginal firm's type $\hat{\theta}$ is decreasing as the policy weight rises, indicating

less efficient firms are dropping out of the industry.

Figure 16.

Marginal θ
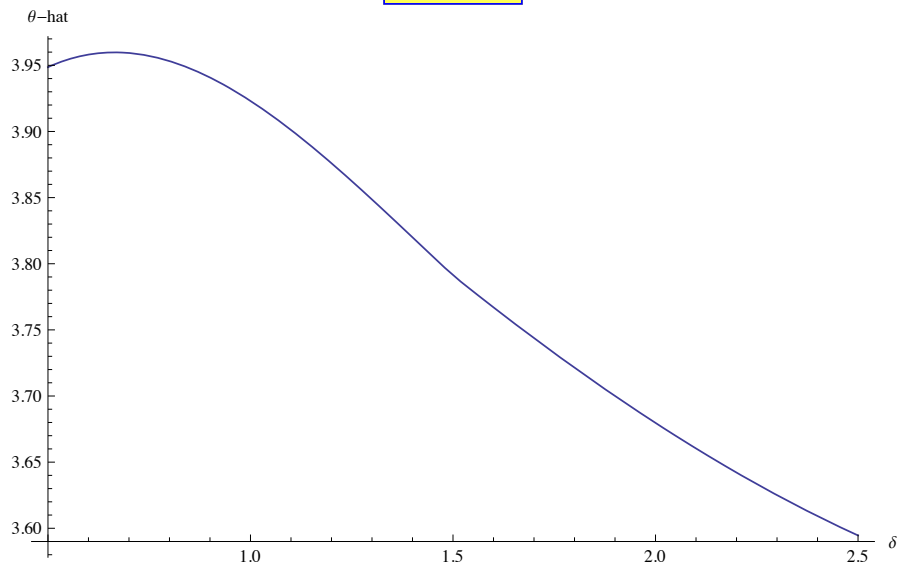
• *Emission and Production: the aggregates*

As firms drop out of the industry, and the remaining firms are producing less, the aggregate emission and production consequently decline.

Figure 17.

Aggregate Emission

Figure 18.

Aggregate Production
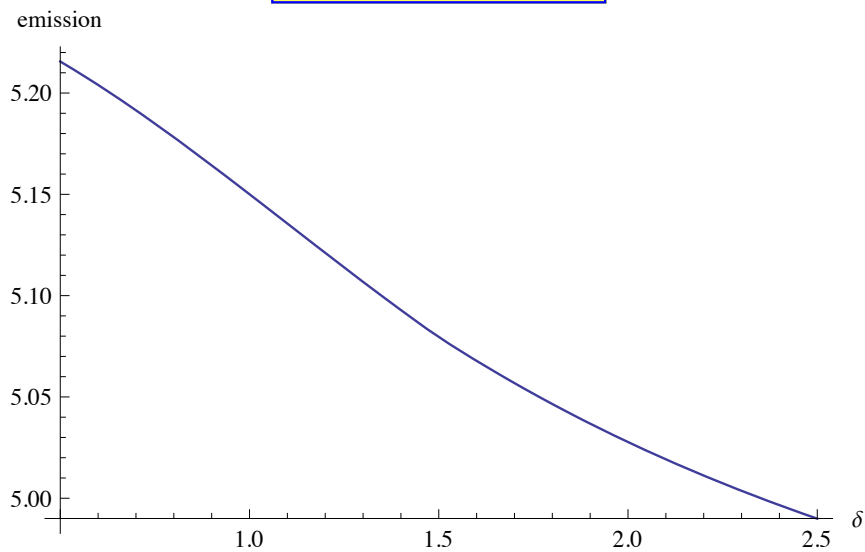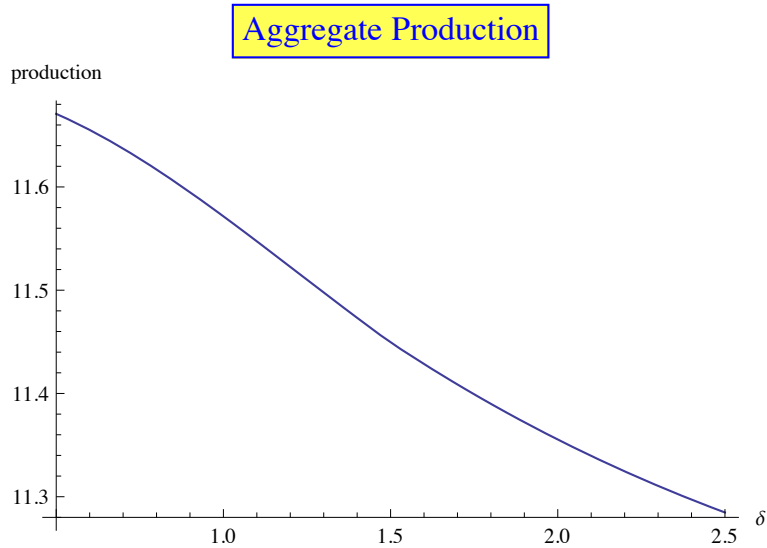
- *Utility and Social Welfare*

As shown in *Proposition 3* in **Chapter 3** (and confirmed by the simulations), when the weight on the penalty increases, an individual firm's utility decreases. Yet since social welfare involves not only the total production but the pollution damage in the economy, social welfare increases due to the reduced emissions when the policy weight increases.

Figure 19.

Social Welfare
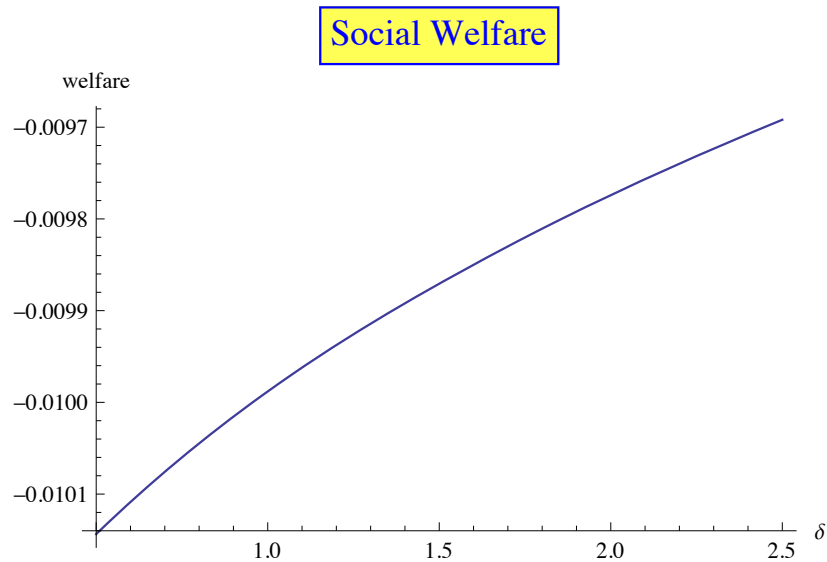
The simulation reveals that as the penalty increases (while the expected fine remains unchanged), firms will drop out of the industry (i.e. the marginal type $\hat{\theta}$ is decreasing as δ increases). On the one hand, aggregate production and pollution both decrease as firms produce less; the industry shrinks. Social welfare, on the other hand, increases as the industry dwindles.[86]

Taking the example of a firm of type-$\theta = 3$, its production, emission and utility all decrease as the severity of the penalty rises. Note that as the weight on the penalty increases, firms actually *cut back* their permit holding along with their pollution level, suggesting it is less expensive for a firm to suppress their production (and thus emission) than to produce the same amount and increase its permit holdings (in order to maintain the same compliance level). Consequently, the equilibrium permit price falls. Though the firm is holding fewer permits, its permit-to-emission ratio rises nonetheless, implying the emission drops more rapidly when the penalty scale increases.

4.2 Monitoring Frequency, π [87]

- *Emission and Production: an individual firm*

---

[86] Note that we impose no cost on adjusting the severity of penalty δ, and since raising δ can promote compliance, as well as depress the production and pollution, social welfare increases as δ goes up. In reality, though, it might incur some social costs (e.g. political complexity, moral debates, etc.) such that a rising δ would have some negative impact on social welfare.

[87] Here we raise the audit frequency alone, while holding the penalty scale fixed.

Similar to the effect of the policy weight, the simulations show that, when audit frequency increases, firms cut back both the emission and production level to avoid the cost of being noncompliant.

Figure 20.

Emission, $\theta = 3$

emission
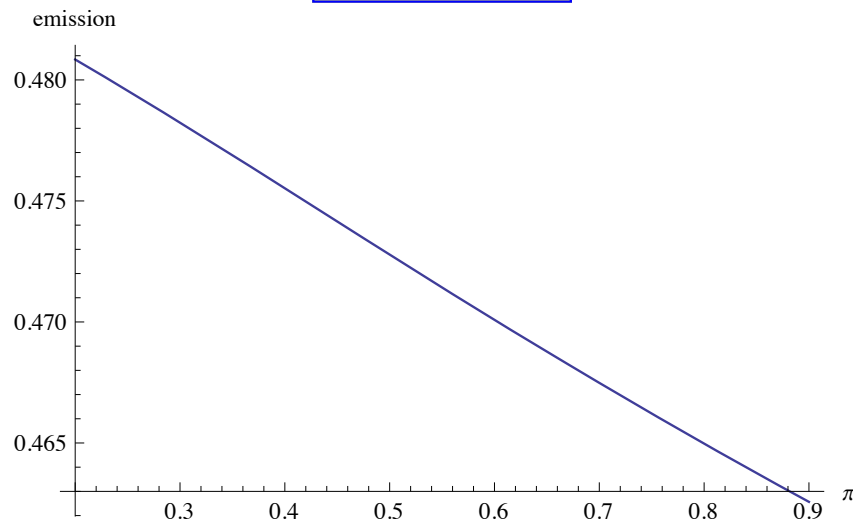
Figure 21.

Production, $\theta = 3$

production
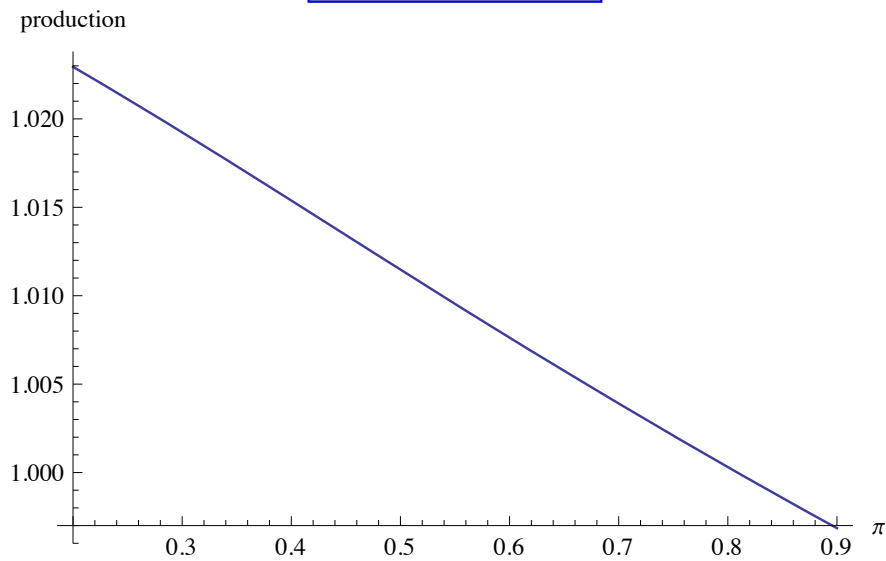
• *Permit Demanded and Permit Price*

In contrast to the policy weight case, the increased audit probability boosts firms' permit holding, and thus pushes the equilibrium permit price higher. This could result from the fact that firms have different sensitivities to the penalty severity and audit frequency. As mentioned in **Chapter 2**, risk averse agents are more sensitive to the severity of the punishment than to the probability of being caught. When the weight on the penalty increases, firms react by cutting back production (and thus emissions) *more than* under the higher chance of being caught, and hence need fewer permits to maintain the same level of compliance. In response to the ascending audit probability, nevertheless, an individual firm's permit demand increases. When the number of permits is fixed, the equilibrium permit price goes up as a result.

Figure 22.

Permit Demanded, $\theta = 3$
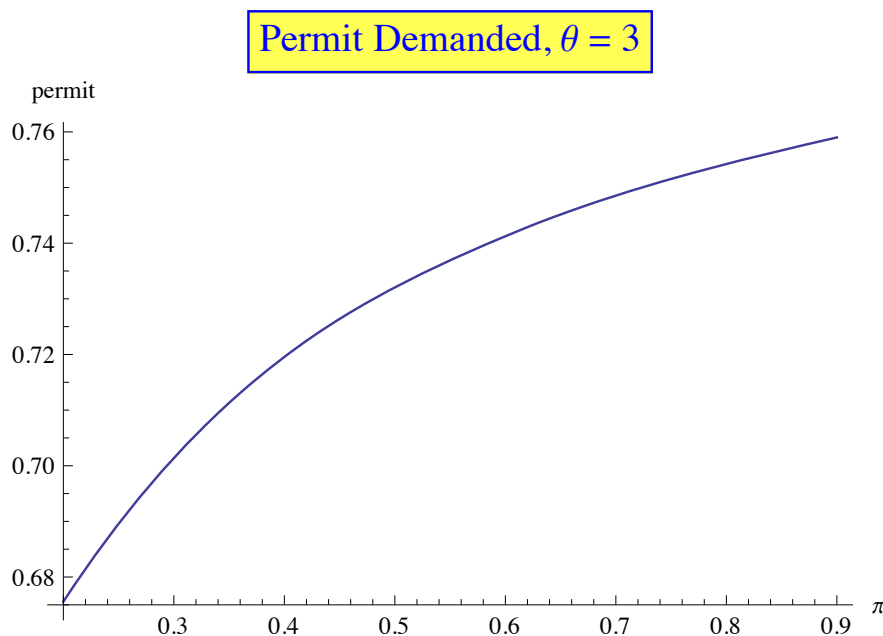
Figure 23.

Permit Price
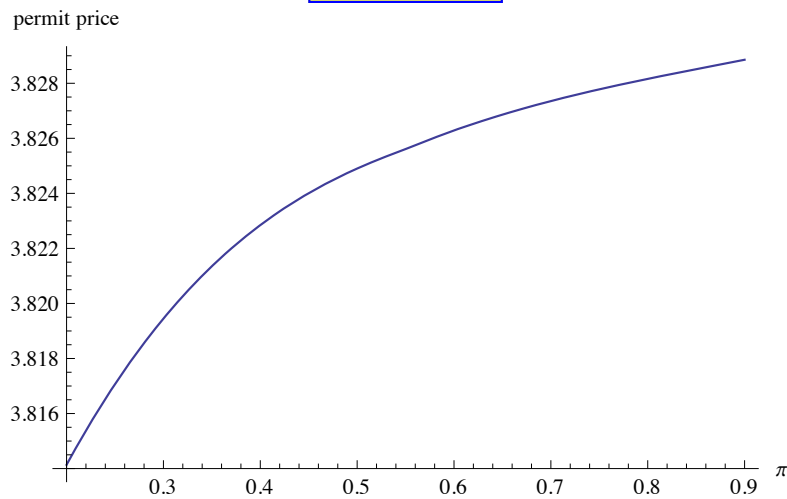
- *Permit-to-pollution ratio* $\hat{\lambda}$

Since the level of permit holding is increasing while the pollution is dropping along with

the rising audit frequency, it follows the compliance ratio $\hat{\lambda}$ is increasing in audit frequency

as well.

Figure 24.

λ–hat, $\theta = 3$
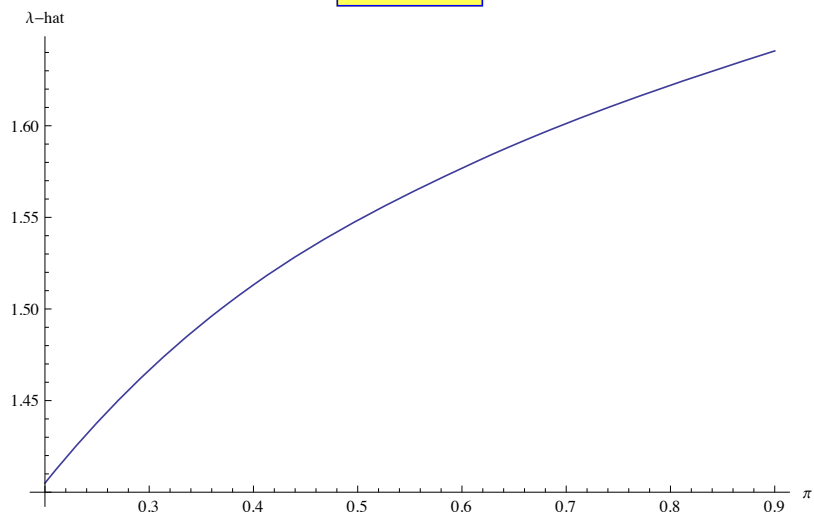
● *Marginal Type $\hat{\theta}$*

Rising permit prices impedes firms' profitability, and hence the marginal firm's type

drops, implying the industry shrinks as in the case of rising policy weight.

Figure 25.

Marginal θ
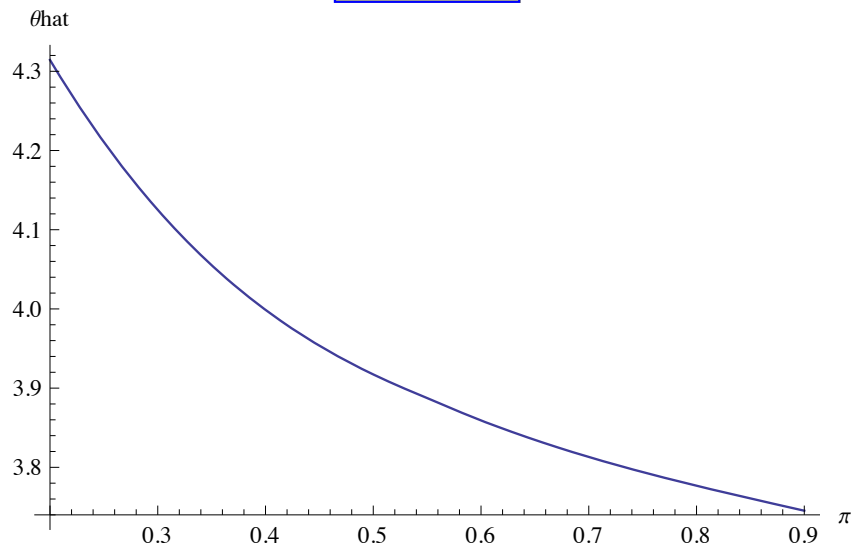
• *Emission and Production: the aggregates*

It follows from shrinking industry and reduced individual production that the aggregate

pollution and production all fall.

Figure 26.

Aggregate Emission

Figure 27.

Aggregate Production
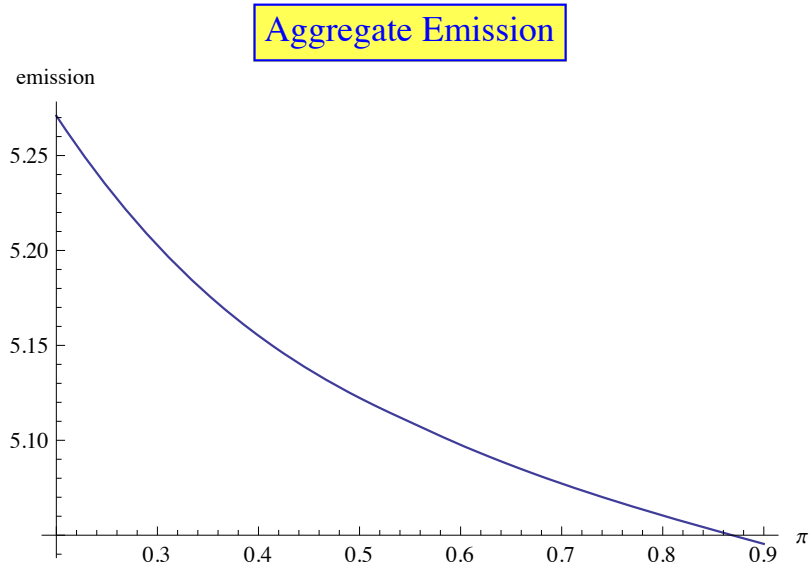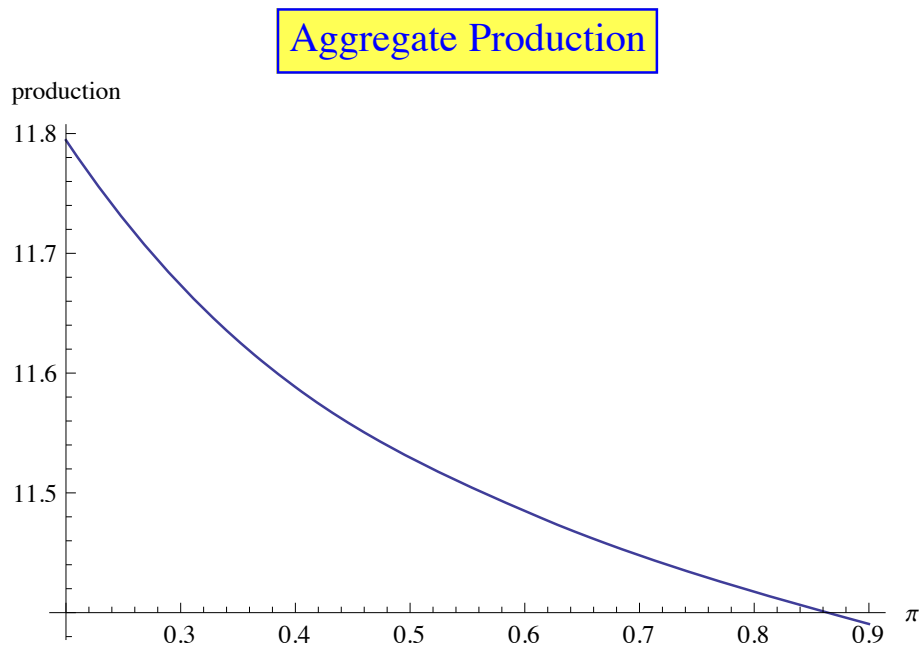
- *Utility and Social Welfare*

Unsurprisingly, an individual firm's utility is decreasing as the audit probability grows. And because we have not imposed any cost on monitoring efforts, social welfare is increasing in the audit frequency.

Figure 28.

Utility, $\theta = 3$
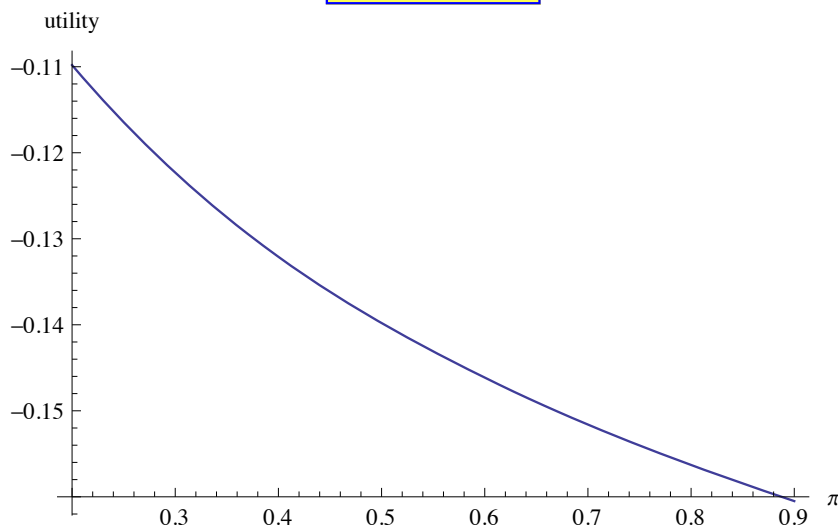
Figure 29.

Social Welfare
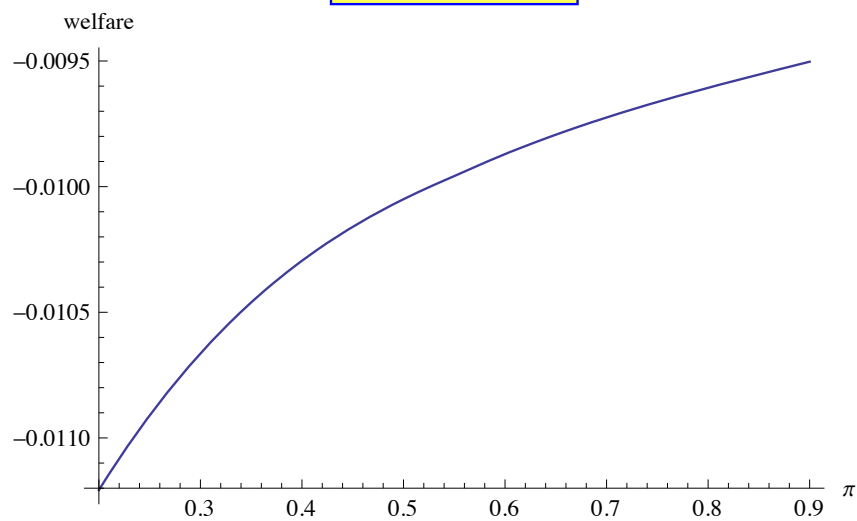
When the government raises the audit frequency alone (without altering the severity of the penalty, unlike in the last case), an individual firm's utility falls, permit holding *increases* and both the emission and production *fall*. This is because, nonetheless, we have not imposed any cost on government monitoring. The best choice for a government in this case, then, is simply to raise the monitoring frequency to its highest possible value.

5. Mean-preserving Spread

Next we show the impact of increased variance of the measurement error on the markets. The following uniform distributions are used. We adopt the same parameter setting as in the previous section to inspect the industry's evolution at equilibrium.

Table 4. Mean Preserving Spread for λ			
Distribution	U[0.9, 1.1]	U[0.5, 1.5]	U[0, 2]
Variance, σ^2	0.0033	0.083	0.33
$f(\lambda)$	5	1	0.5

• *Permit Price and Marginal Type of Firm*

First we compute the optimal permit supply for each spread to determine the equilibrium permit price and the industry size.[88] Optimally, the government is handing out more permits as the measurement error grows, and thus the equilibrium permit price falls. Lower permit

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[88] For this section, we first compute the optimal permit supply for each spreads, given the parameterization in **Table 1**. The following comparative statics is assessed while fixing the permit supply at these optimal values.

price entices more firms to join the industry, such that the marginal firm's type rises as the risk increases.[89]

Table 5. Permit Market and Industry Equilibrium				
Distribution	Certainty	U[0.9, 1.1] $\sigma^2 = 0.0033$	U[0.5, 1.5] $\sigma^2 = 0.083$	U[0, 2] $\sigma^2 = 0.33$
Permit Supply	5.266	5.373	7.214	9.432
Permit Price	7.389	6.781	5.042	3.822
Marginal Type $\hat{\theta}$	3.487	2.958	3.750	3.921

Figure 30.
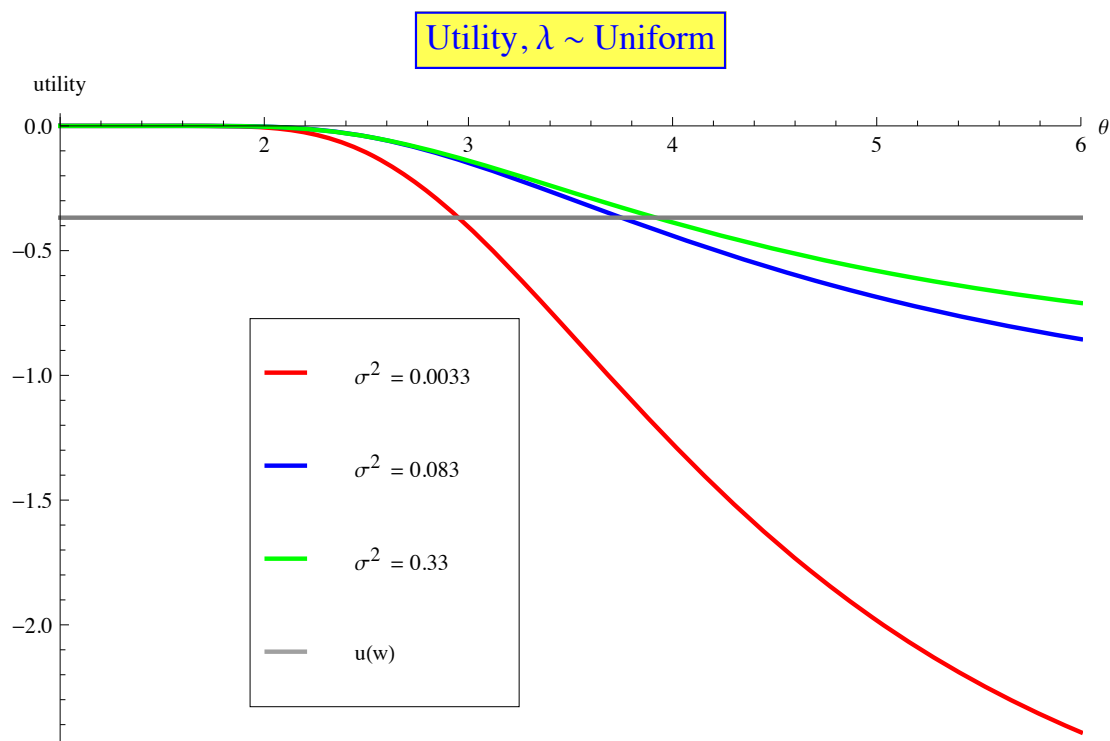
Nevertheless, we discover that as the uncertainty increases, the equilibrium permit price decreases: since the government is using the number of permits as a means to mitigate the

_____

[89] The grey line in the graph below is the marginal utility. For firms of higher type value than the marginal type the value of utility is irrelevant.

risk in addition to environmental damage control, optimally then, the government is issuing more permits in response to higher risk. In **Figure 31**, we see that the optimal permit supply (dashed vertical lines, in accordance with the curves of the same color) moves outward as the uncertainty grows.

Figure 31.
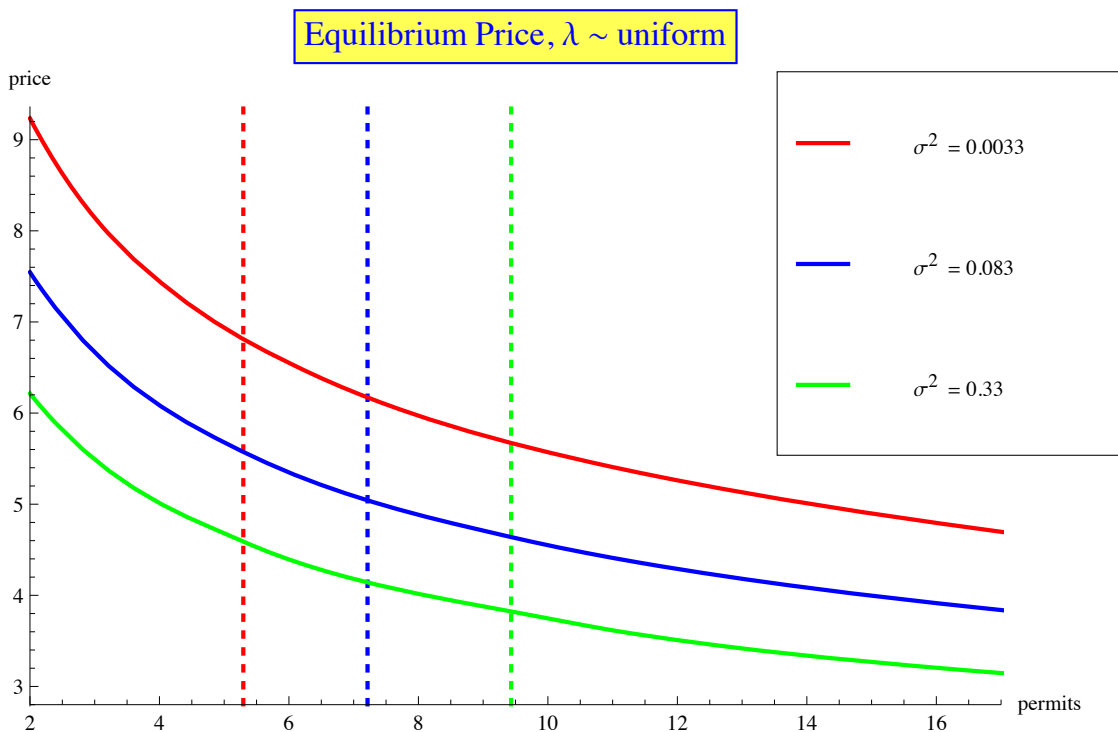
- *Production q and Pollution e*

The production and pollution, for both an individual firm and the industry, are not monotonic in risk. To see this, we take the firm of type $\theta = 3$ as the illustration sample.

For a firm of type $\theta = 3$, its production and emissions both oscillate in the range of simulations with similar patterns, and there seems no apparent rules for the decrease or increase with respect to the growing risk. These results are shown in the graphs below:[90]

Figure 32.
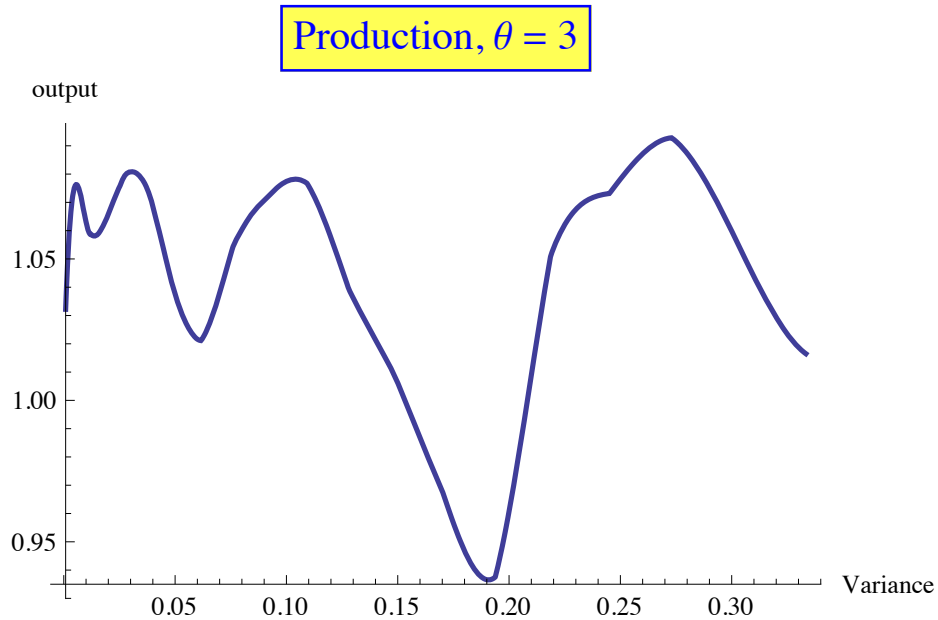
Production, $\theta = 3$

Figure 33.

Emission, $\theta = 3$
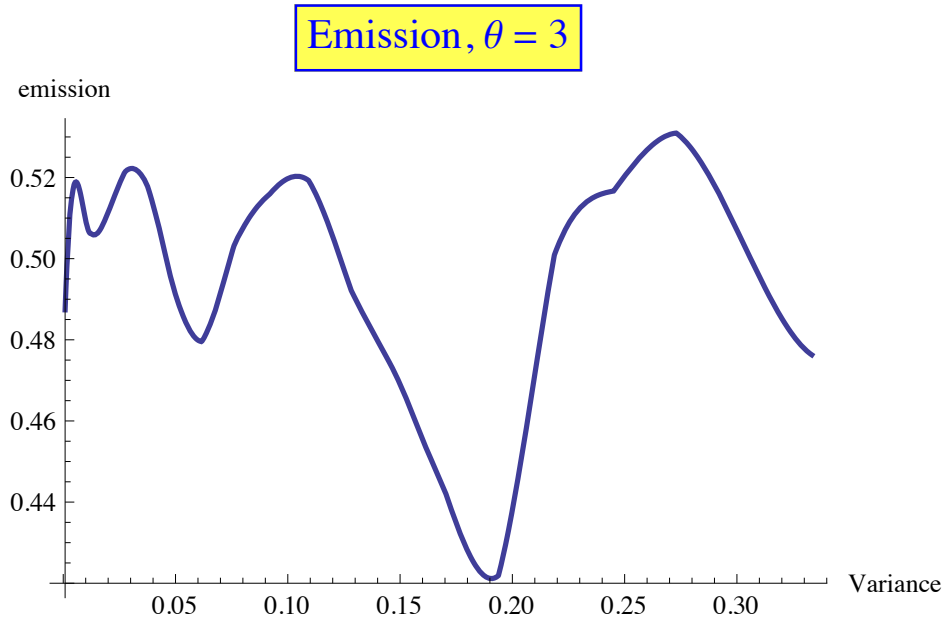
From the simulated industry output and pollution in **Table 6** below, we can see that there is no sure direction of changes in these aggregate variables. Consider the industry is expanding as the uncertainty rises, whether the aggregate output/pollution will rise or fall depends on the combined effects of two forces: as new entrants raise the capacity of the industry, the incumbents' production/emission decisions are undetermined, and thus the combined scale of change in the aggregate production/pollution is ambiguous.

Table 6. Production and Emission			
Distribution	U[0.9, 1.1] $\sigma^2 = 0.0033$	U[0.5, 1.5] $\sigma^2 = 0.083$	U[0, 2] $\sigma^2 = 0.33$
Production	11.529	11.804	11.550
Emission	5.256	5.324	5.134

● *Permit-to-pollution Ratio, $\hat{\lambda}$*

Though we cannot say the emission falls monotonically as the variance of error increases, the degree of compliance in terms of permit-to-pollution ratio, nevertheless, is increasing as the risk grows, suggesting a firm's demand for permits must be more sensitive to the growing risk. Furthermore, here we still observe that more efficient firms have higher $\hat{\lambda}$:

Figure 34.
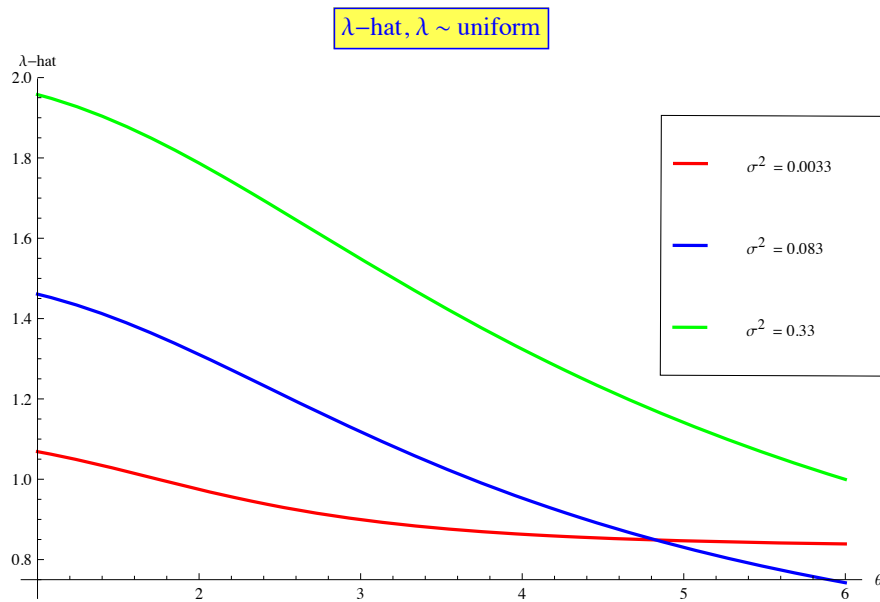
● *Social Welfare*

Social welfare increases as the error grows more dispersed (comparing the 3 distributions). In a deterministic world, the welfare ranking is actually higher than that of the uncertainty cases, yet once we're in the world with risk, welfare drops sharply, and then climbs up as uncertainty grows.

The following 3 graphs depict the equilibrium permit price and the marginal type, and then the welfare with respect to changes in error variances. Note that though the equilibrium permit price and the size of industry seem monotonic with respect to the variance, welfare is not. In fact, like the case of the individual firm's choices of production and emission, it oscillates as the risk increases, and exhibits a tendency of growing amplitude as the variance becomes larger. One possible explanation could be that as the government adjusts the number of permits in response to the increased risk, the equilibrium permit price decreases, drawing more entrepreneurs. And since the equilibrium permit price changes more drastically than the pollution, the benefit of decreased permit price could out-weight the negatives resulting from growing uncertainty.[91]

Table 7. Social Welfare			
Distribution	U[0.9, 1.1] $\sigma^2 = 0.0033$	U[0.5, 1.5] $\sigma^2 = 0.083$	U[0, 2] $\sigma^2 = 0.33$
Welfare	-0.0121	-0.0116	-0.0102

_____

[91] See Footnote 90.

Figure 35.

Equilibrium Permit Price

price

Figure 36.

Optimal Marginal Type

cutoff θ
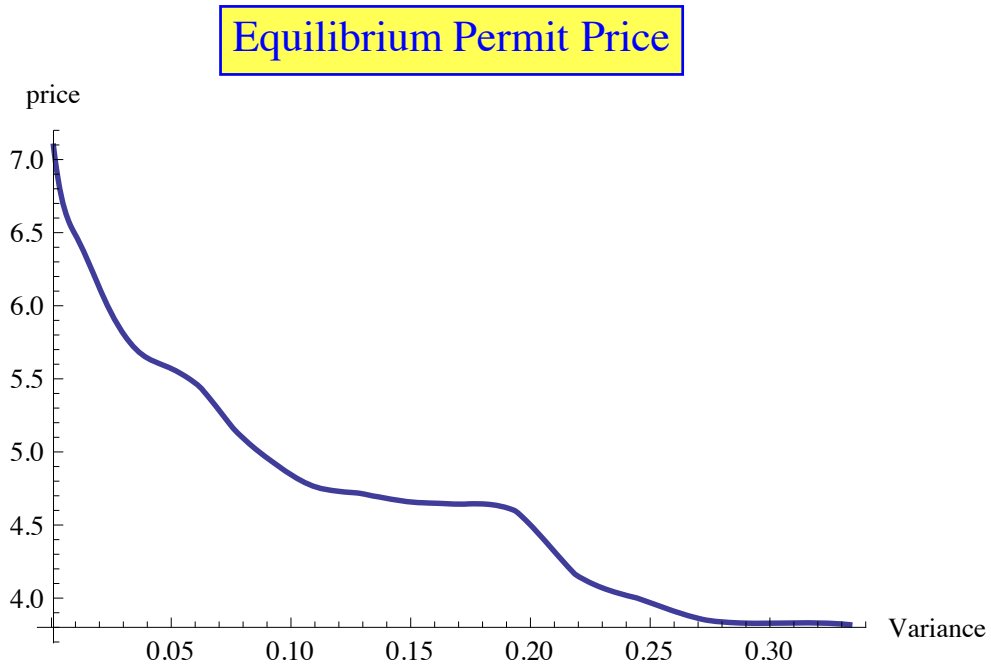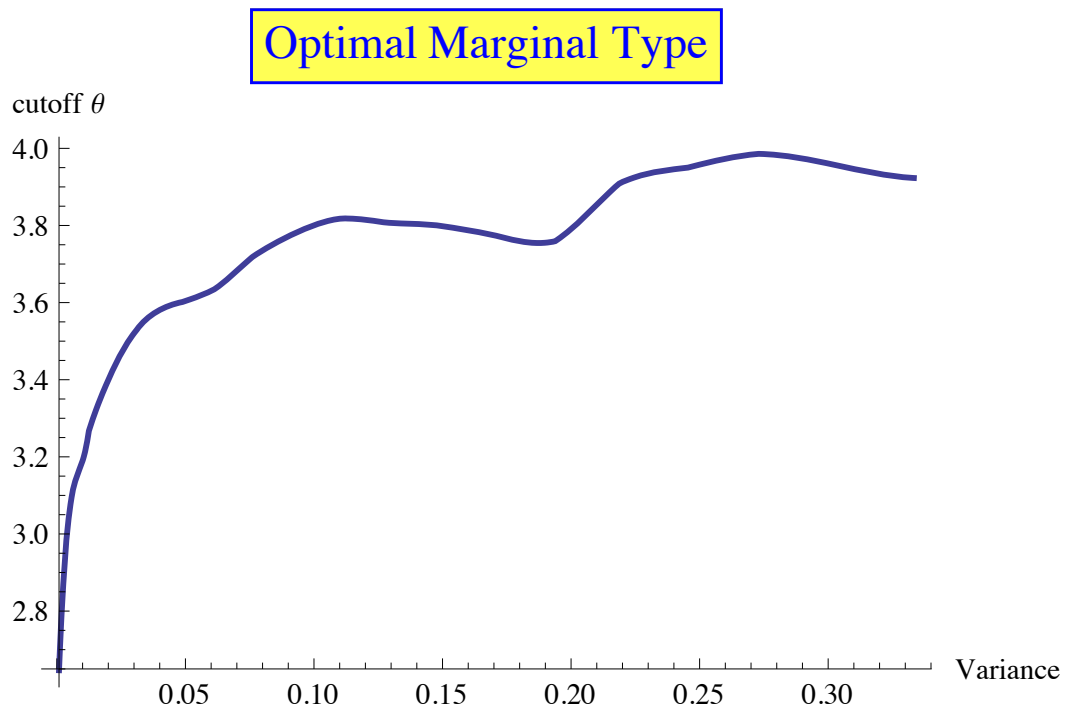
Figure 37.

Welfare
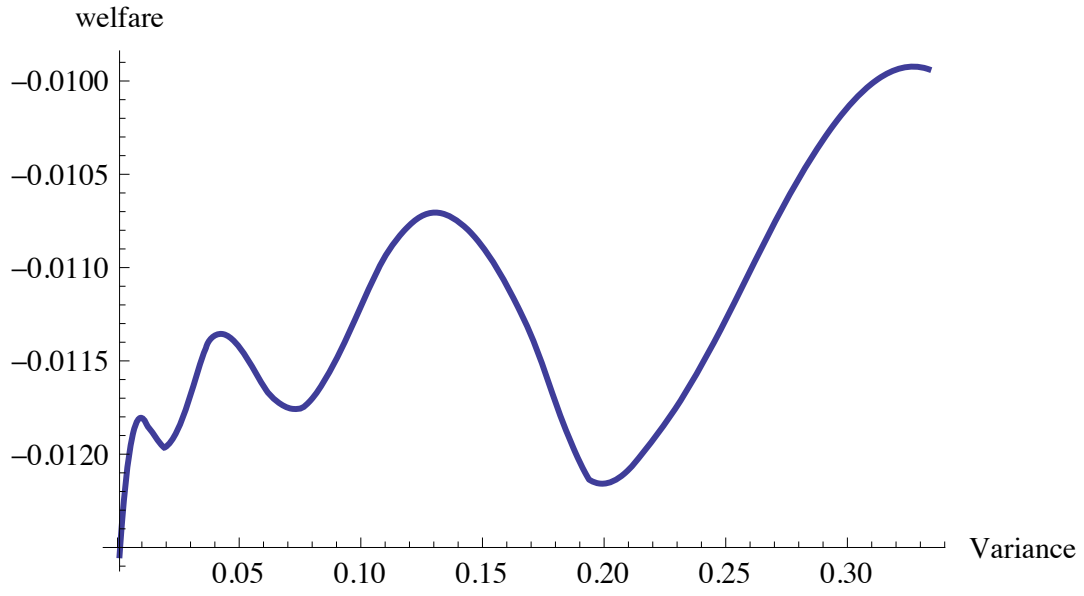
6. Robustness Test

To see how robust our model is under a different error term distribution, we assess the case with an error of log-normal distribution, truncated on the same interval $[0,2]$ [92], in contrast to the uniform distribution case. [93]

6.1 Industry Equilibrium

As in the uniform error case, we first determine the optimal permit supply that maximizes social welfare, and then assess the impacts of parameters on the industry in aggregate, and individual firms' behavior.

From the graphs below, we see that the social welfare attains its maximum around $L \approx 10$. The equilibrium permit price decreases as permit supply goes up.

The equilibrium values of the variables are summarized in Table 8:

Table 8. Optimal Variable Values under Log-normal Distribution					
Permit Supply	Permit Price	Production	Emission	Marginal $\hat{\theta}$	Welfare
9.369	3.831	11.695	5.217	4.166	-0.0109

---

[92] In the simulation we use the normal distribution $N(\mu = 0, \sigma = 0.5)$ for the log-normal transformation of the error λ.

[93] Robustness check of the damage coefficient α is shown in Appendix 4.

Figure 38.

Social Welfaren, $\lambda \sim$ truncated LN

welfare

Figure 39.

Equilibrium Price, $\lambda \sim$ truncated LN

price
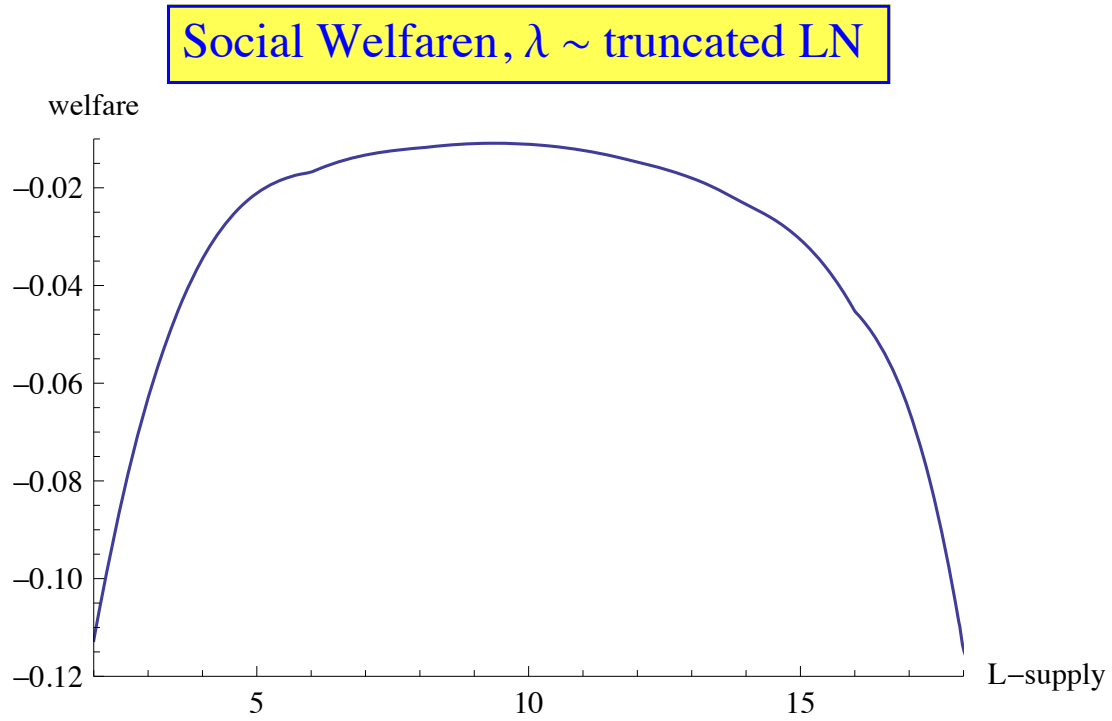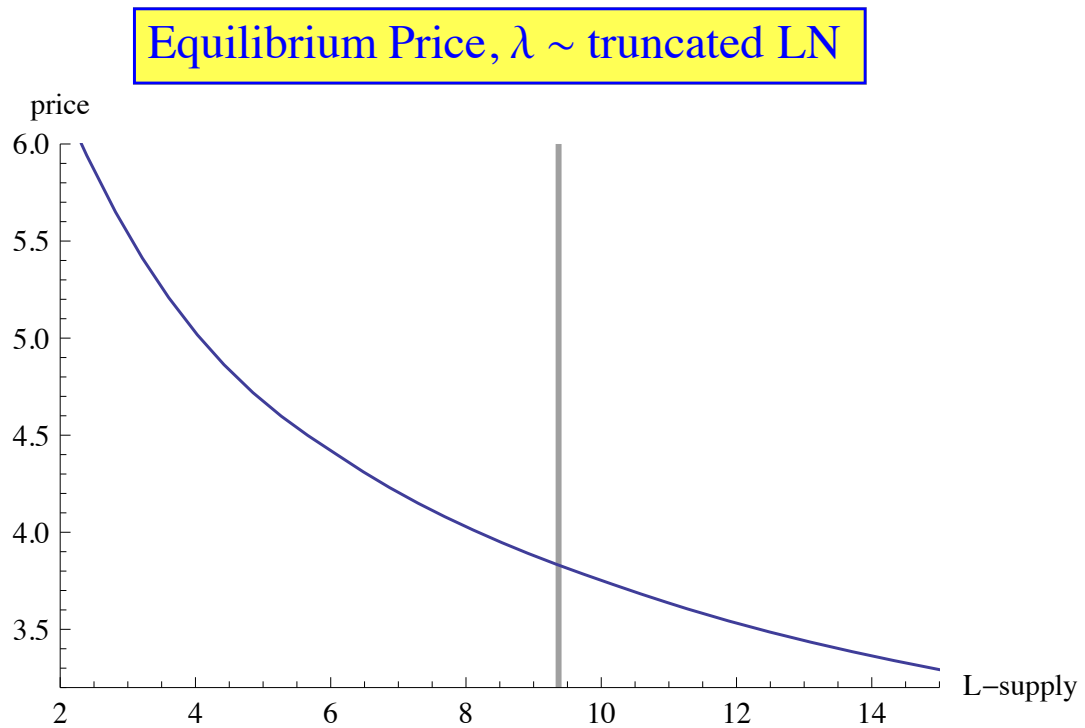
The equilibrium and the marginal firm's type are determined through the market equilibrium conditions:

Figure 40.

Aggregate Permit Demand, $\lambda \sim$ truncated LN

Figure 41.

Utility, $\lambda \sim$ truncated LN
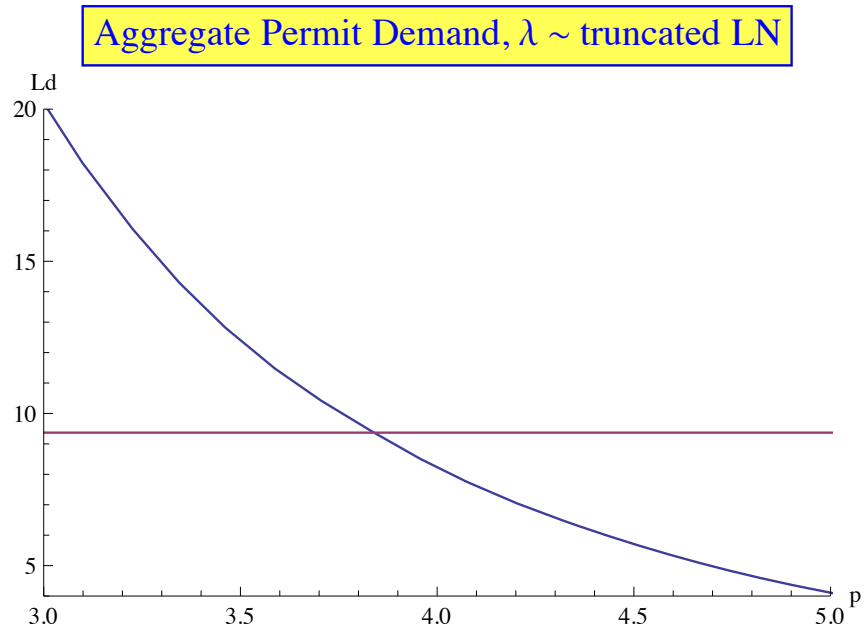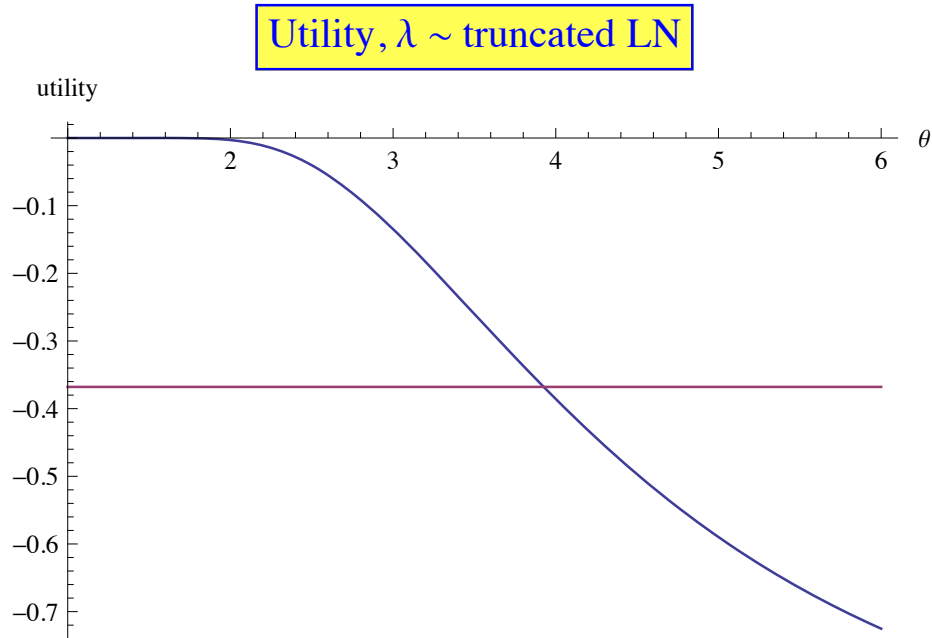
Through simulations, we show that firms' behavior under log-normal error exhibits the same pattern as that under the uniformly distributed error.

6.2 Individual Firms Optimal Choices

- *Production q and Pollution e*

As in the uniformly-distributed error case, the simulations also confirm that firms' production and emission decreases as their efficiency decline.

- *Permit Demanded ℓ and Permit-to-pollution Ratio, $\hat{\lambda}$*

Because a more efficient firm produces more, it also tends to hold more permit. The permit-to-pollution ratio, $\hat{\lambda}$, is hence larger for more efficient firms. The gray line marked the marginal firms' type.

Figure 42.

$\lambda-hat, \lambda \sim$ truncated LN
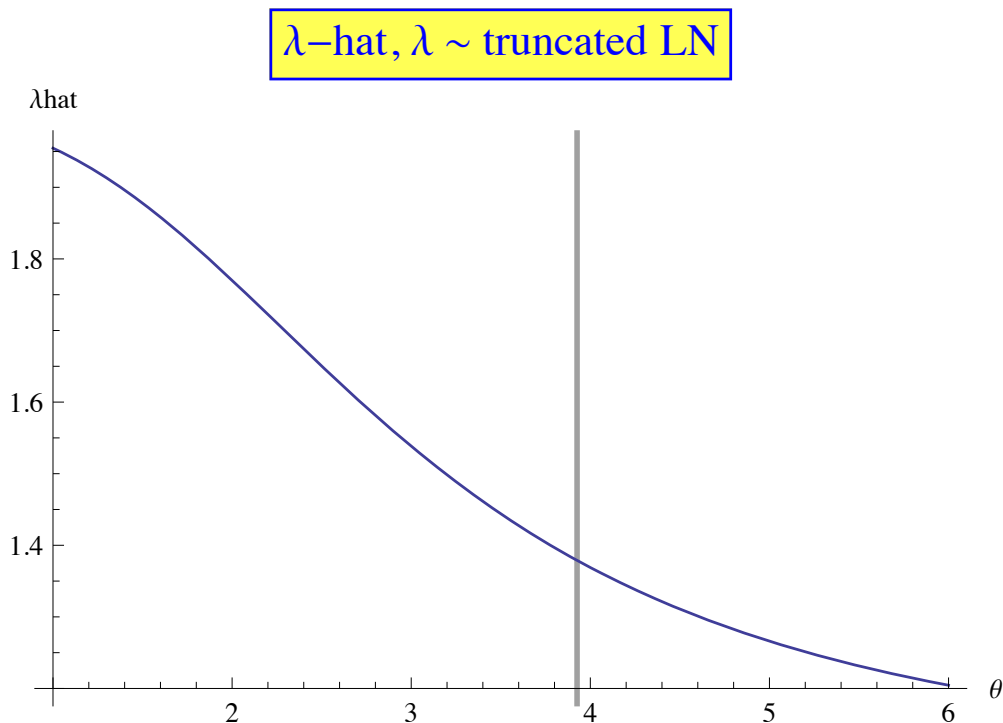
We then assess the impacts of the policy instruments on the industry and individual firms, when the measurement error follows the log-normal distribution.[94]

6.3 Policy Weight [95]

- *Emission and Production: an individual firm*

Individual emission and production both fall as the weight on penalty increases.

Figure 43.

Emission, $\theta = 3$

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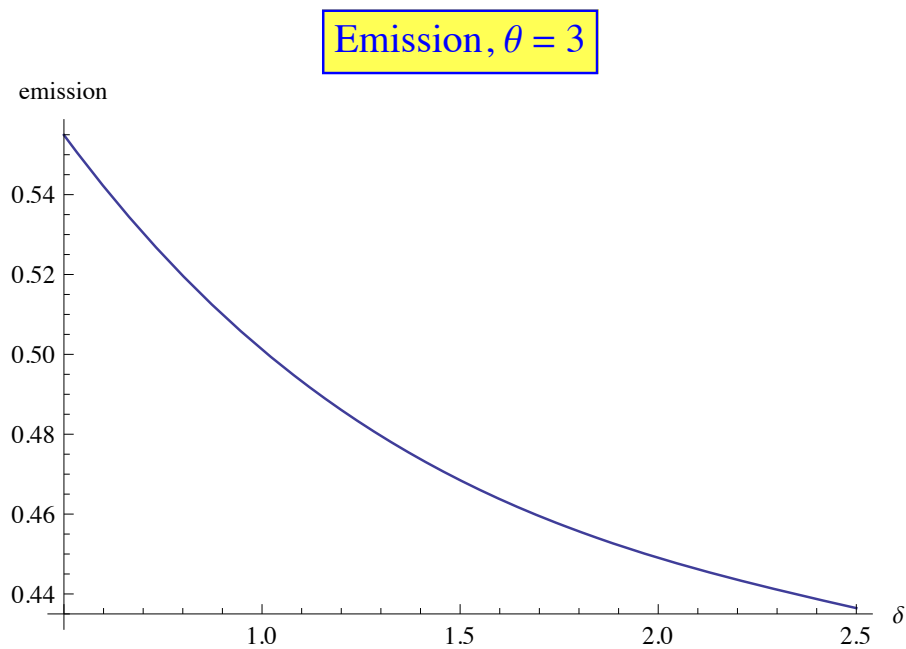
[94] Again, we use the firm whose type $\theta = 3$ as the individual example. The comparative statics is done when fixing the number of permits at its optimal value.

[95] Since the trends are similar to that of the uniform error, we do no spend much on the interpretations again.

Figure 44.

Production, $\theta = 3$
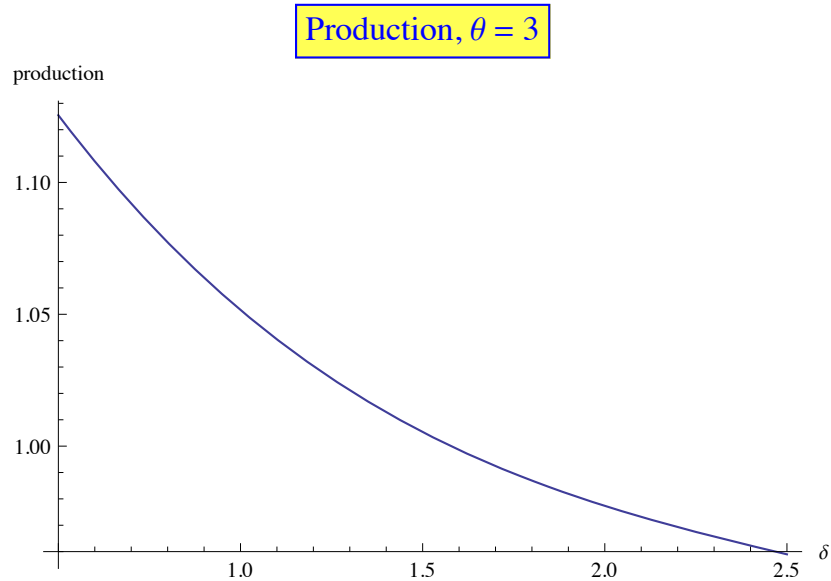
• *Permit Demanded and Permit Price*

The individual firm's permit demand falls as the weight on the penalty increases. The simulations also show that, with log-normally distributed error, the permit price is still downward-sloped along δ.

Figure 45.

Permit Demanded, $\theta = 3$

Figure 46.

Permit Price, Lognormal Error
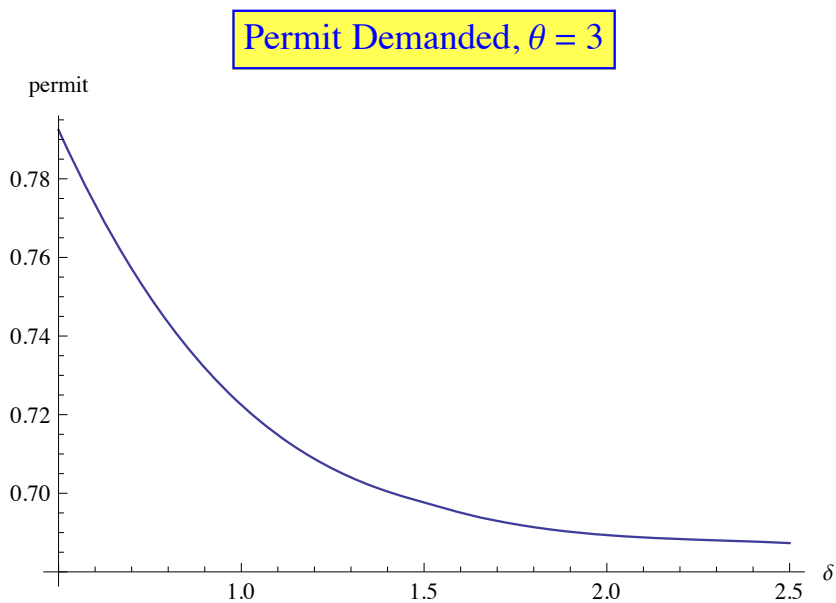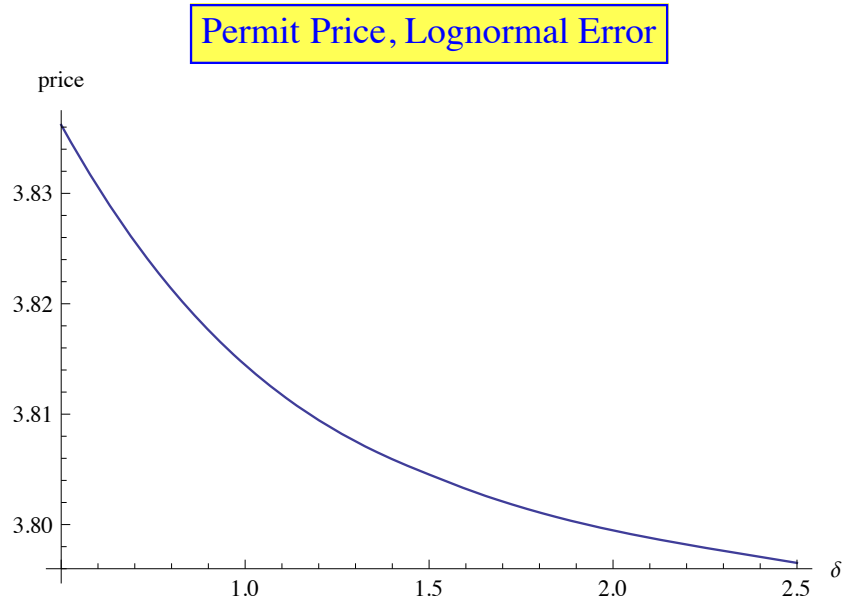
- *Permit-to-pollution ratio $\hat{\lambda}$ and the Marginal Type $\hat{\theta}$*

Though firms cut their permit holding in the course of rising policy weight, the degree of compliance in terms of permit-to-pollution ratio $\hat{\lambda}$, is still rising in the severity of penalty.

The marginal firm's type $\hat{\theta}$ is decreasing as the policy weight rises, indicating less efficient firms are dropping out of the industry as the penalty scale aggravates.

Figure 47.
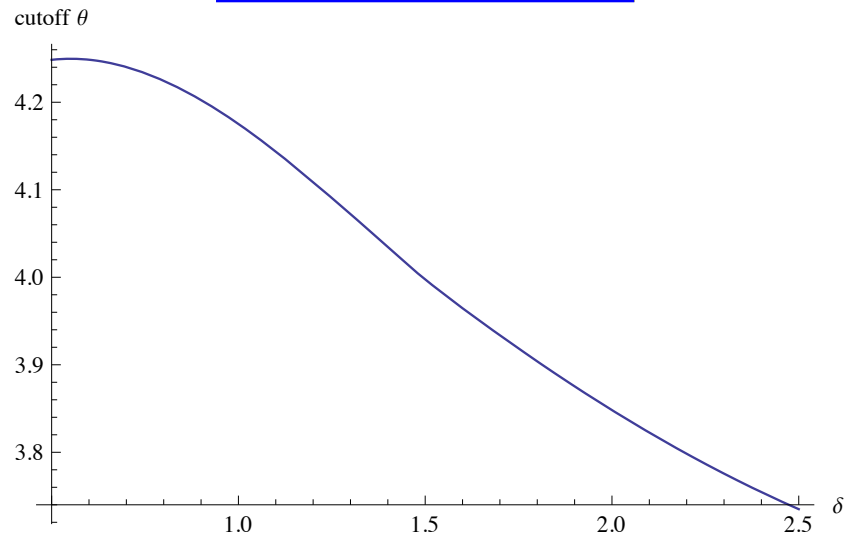
Figure 48.

• *Emission and Production: the aggregates*

Since firms are cutting back their emission and hence the production, and less efficient firms are dropping out of the industry, the aggregates both exhibits downward-sloping trends along δ.

Figure 49.

Aggregate Emission, Lognormal Error

Figure 50.

Aggregate Production, Lognormal Error
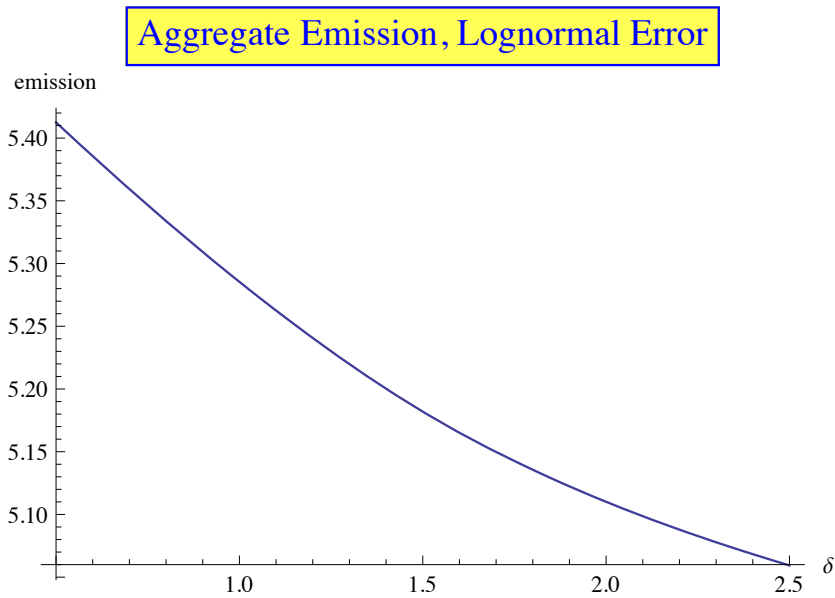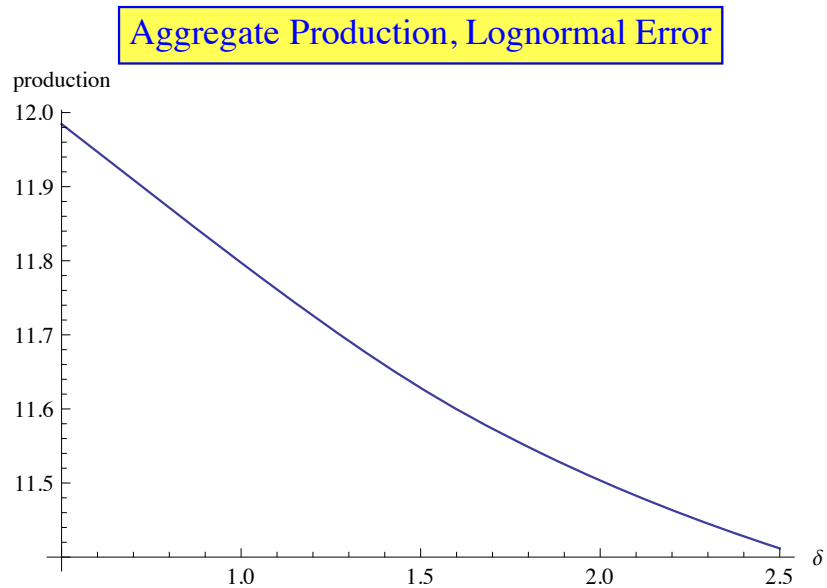
- *Utility and Social Welfare*

Individual firm's utility is decreasing in the policy weight δ . The social welfare, however, is increasing in δ .

Figure 51.

Utility, $\theta = 3$
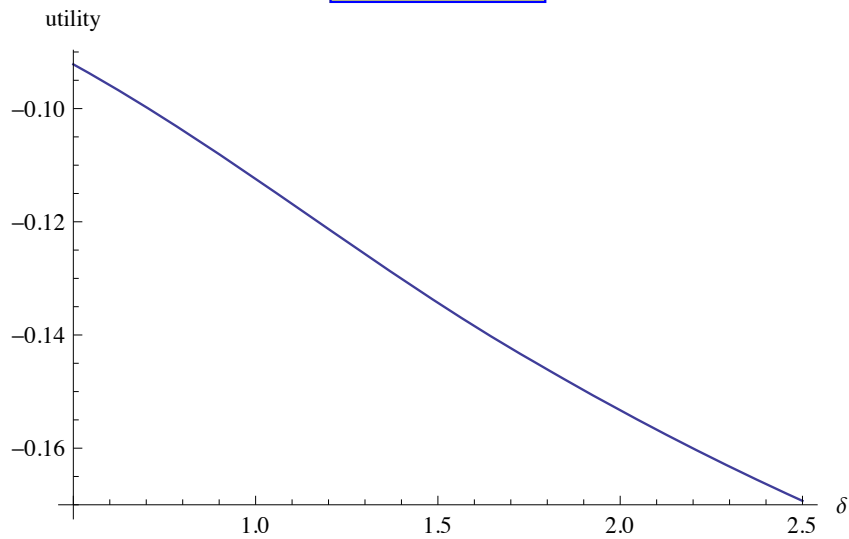
Figure 52.

Social Welfare, Lognormal Error
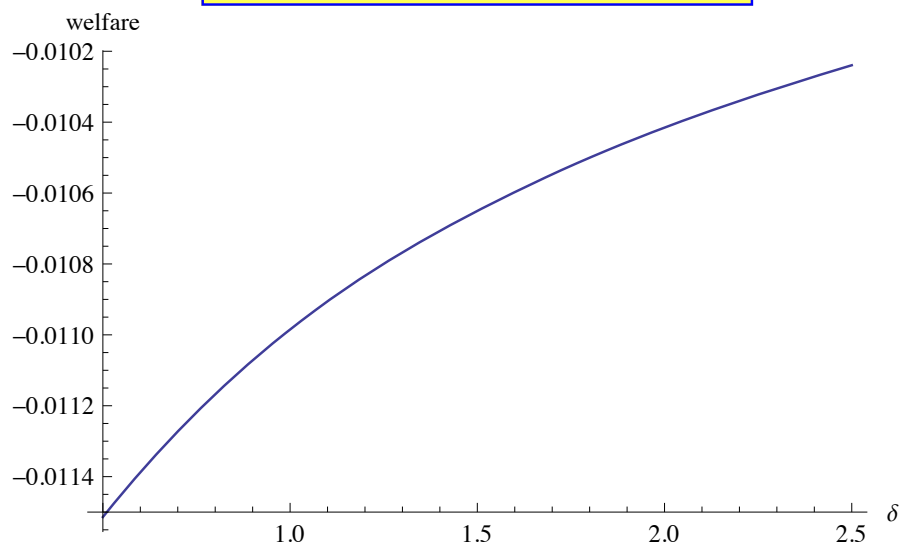
6.4 Monitoring Frequency, π [96]

- *Emission and Production: an individual firm*

Individual emission and production both fall as weight on penalty increases.

Figure 53.

Emission, $\theta = 3$

Figure 54.

Production, $\theta = 3$

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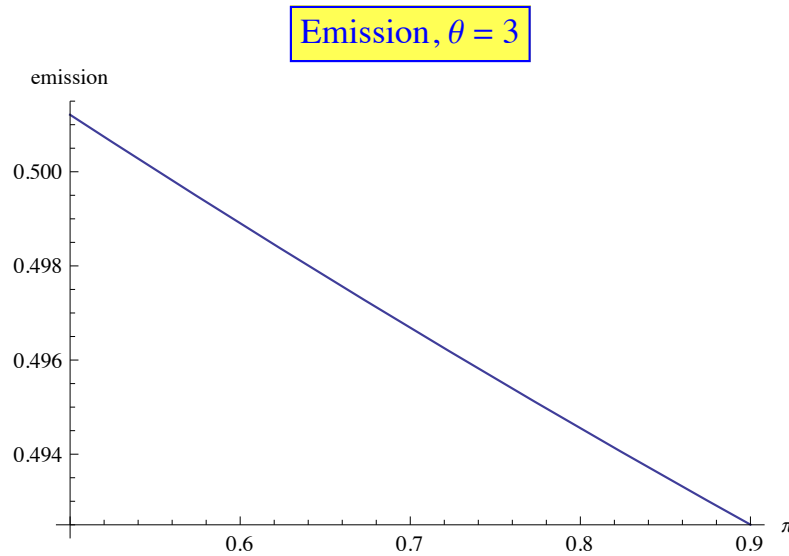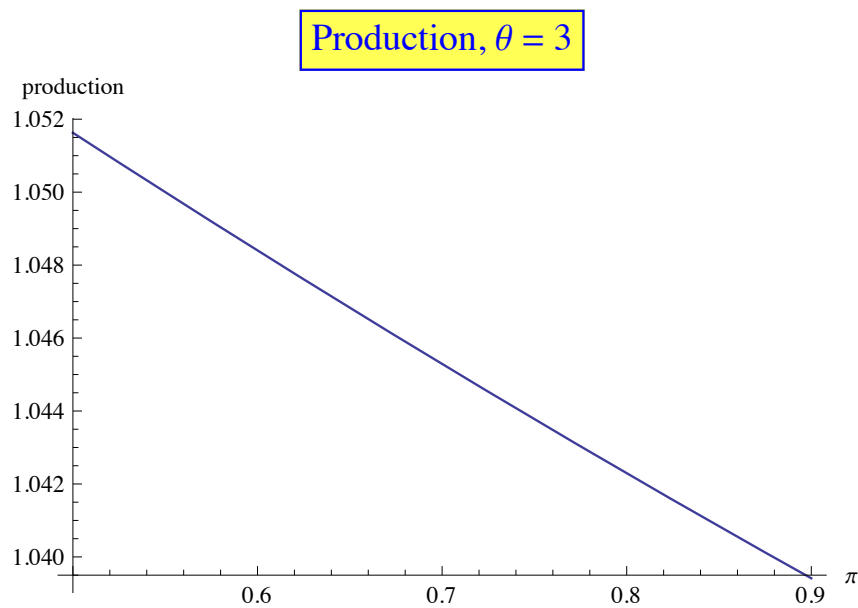
[96] To ensure the marginal type $\hat{\theta}$ lies in the set of $[\underline{\theta}, \overline{\theta}]$, we set the audit frequency in the range of $[0.5, 1)$.

- *Permit Demanded and Permit Price*

In contrast to the policy weight case, increased audit probability boosts firms' permit holding, and thus pushing the equilibrium permit price higher.

Figure 55.

Permit Demanded, $\theta = 3$

Figure 56.

Permit Price, Lognormal Error
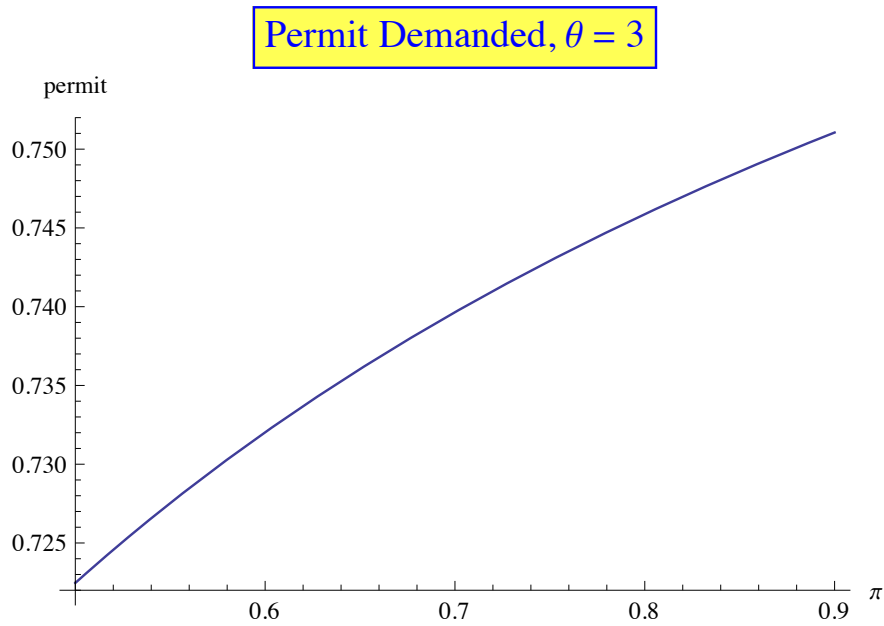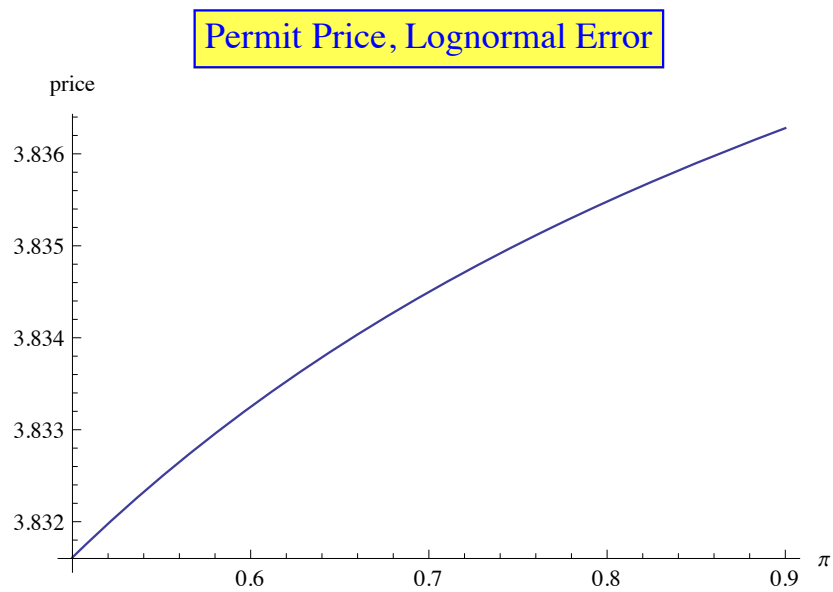
- *Permit-to-pollution ratio $\hat{\lambda}$ and the Marginal Type $\hat{\theta}$*

The degree of compliance is increasing in the audit frequency, as firms are cutting their pollution and increasing their permit holding at the same time. The marginal firm's type $\hat{\theta}$ is decreasing as the audit frequency rises. Both results are consistent with the uniform error case.

Figure 57.

$\lambda-\text{hat}, \theta = 3$

Figure 58.

Marginal θ, Lognormal Error
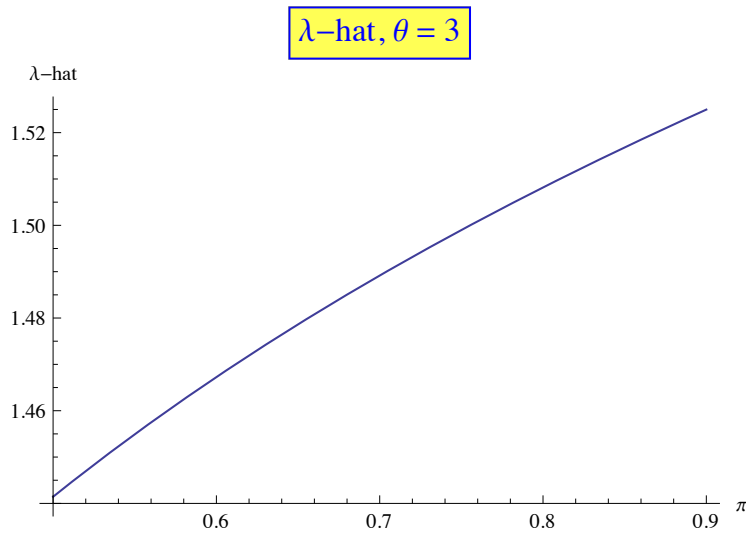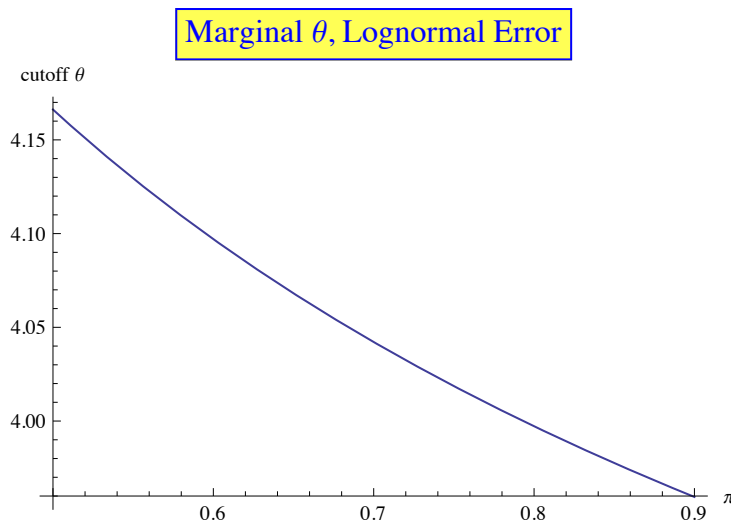
• *Emission and Production: the aggregates*

Similar to the responses to rising policy weight, when the audit rate increases, individual firms' production/emission drop. The aggregate output/pollution are further reduced as firms are leaving the industry.

Figure 59.
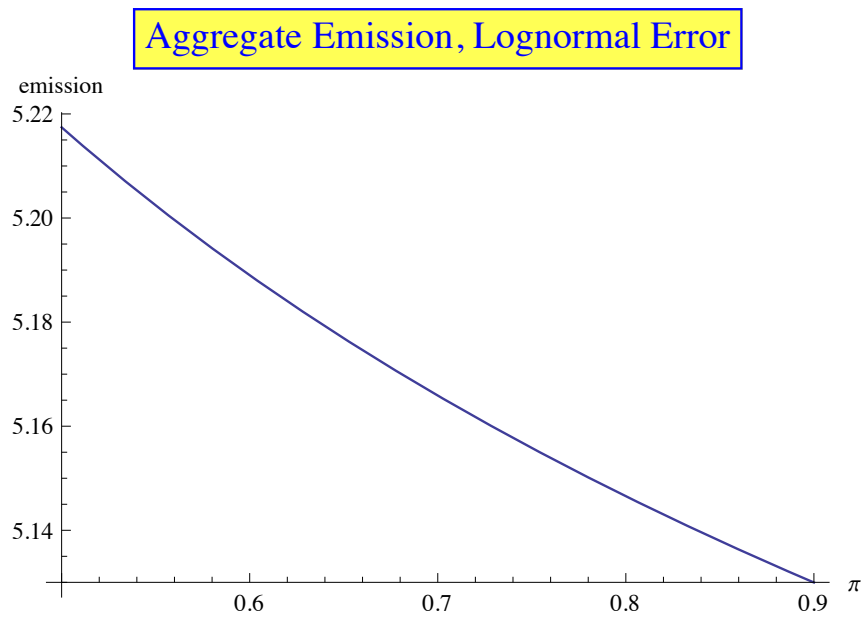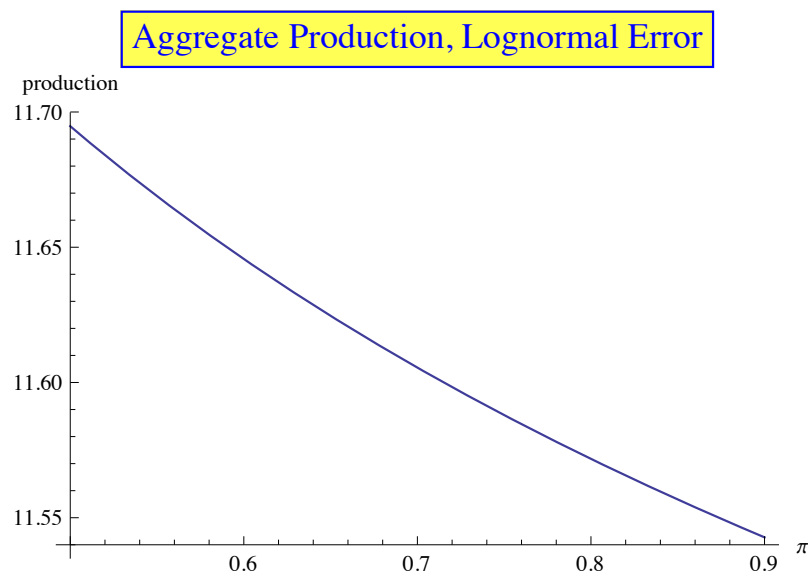
Aggregate Emission, Lognormal Error

Figure 60.

Aggregate Production, Lognormal Error

- *Utility and Social Welfare*

Individual firm's utility is decreasing in the audit frequency π. The social welfare is increasing as the audit frequency rises.
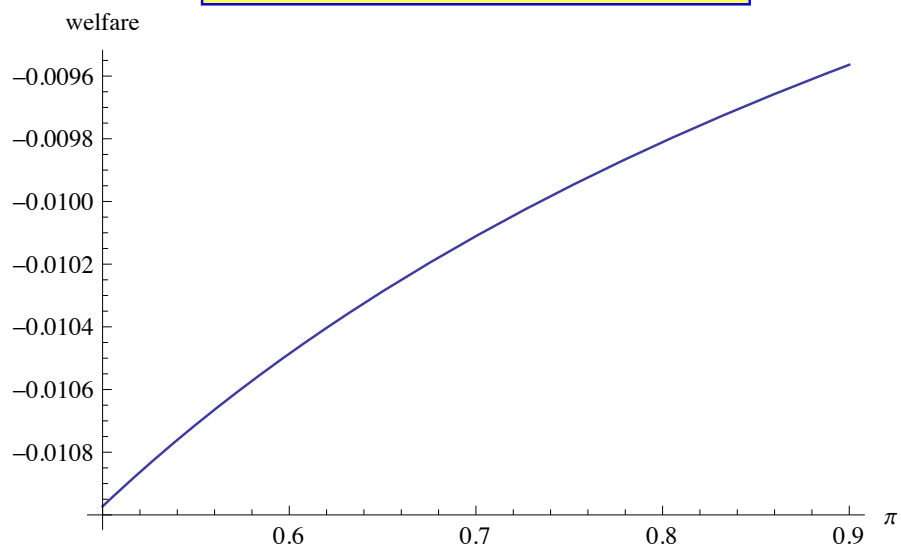
Figure 61.

Utility, $\theta = 3$
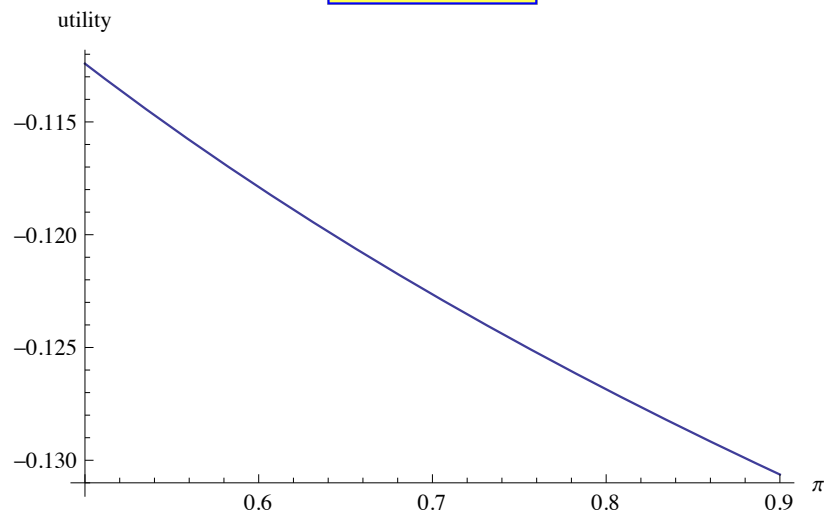
Figure 62.

Social Welfare, Lognormal Error

The robustness test shows that in general, firms and the industry both exhibit similar behavioral patterns in response to the adjustments of policy instruments to that under a uniform error case: downward-sloping equilibrium permit price along the penalty scale, and increased permit price as the audit frequency increases; social welfare increases when the government tightens its regulations.

7. Best Policy

We then explore the best policy set for the government. That is, the optimum when the government is choosing the policy weight and the permit supply simultaneously.

In search of the best instrument set, we find that when the government can adjust both the policy weight and the permit supply, the best policy is to set the weight on penalty to as large as $\delta \approx 4.6$, while the permit supply remains around $L^* \approx 11$.[97]

The following graph depicts the welfare change in terms of δ and the number of permits.

---

[97] In this simulation, we went back to the uniform error: $\lambda \sim U[0,2]$. Note that in the previous simulations for the comparative statics, we set the upper bound the policy weight δ to 2.5. But in the search of optimal policy, we broaden the range of possible penalty scale.

Figure 63.
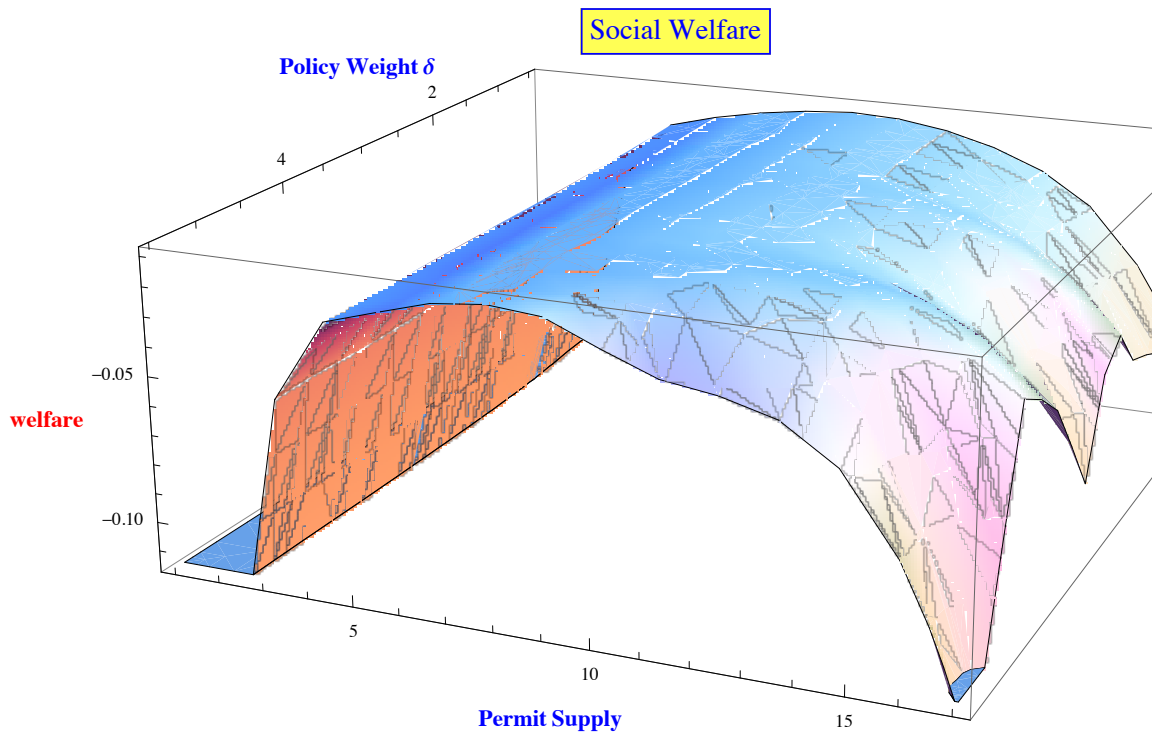
As permit supply and the policy weight are changing simultaneously, the equilibrium price moves on a smoother surface:

Figure 64.
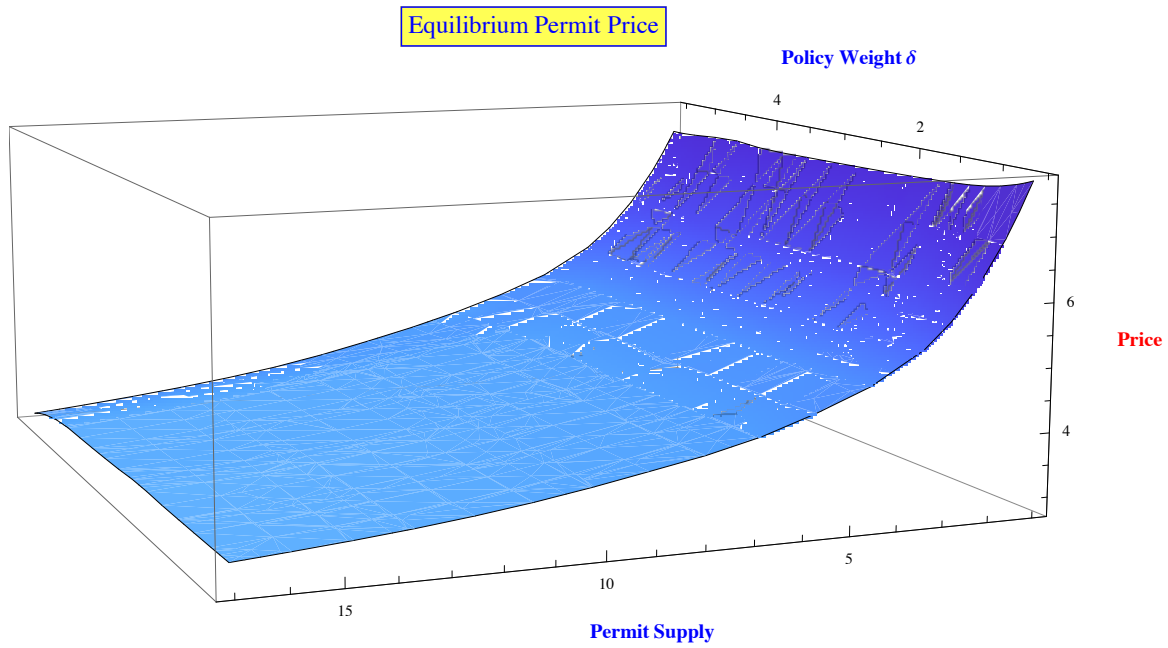
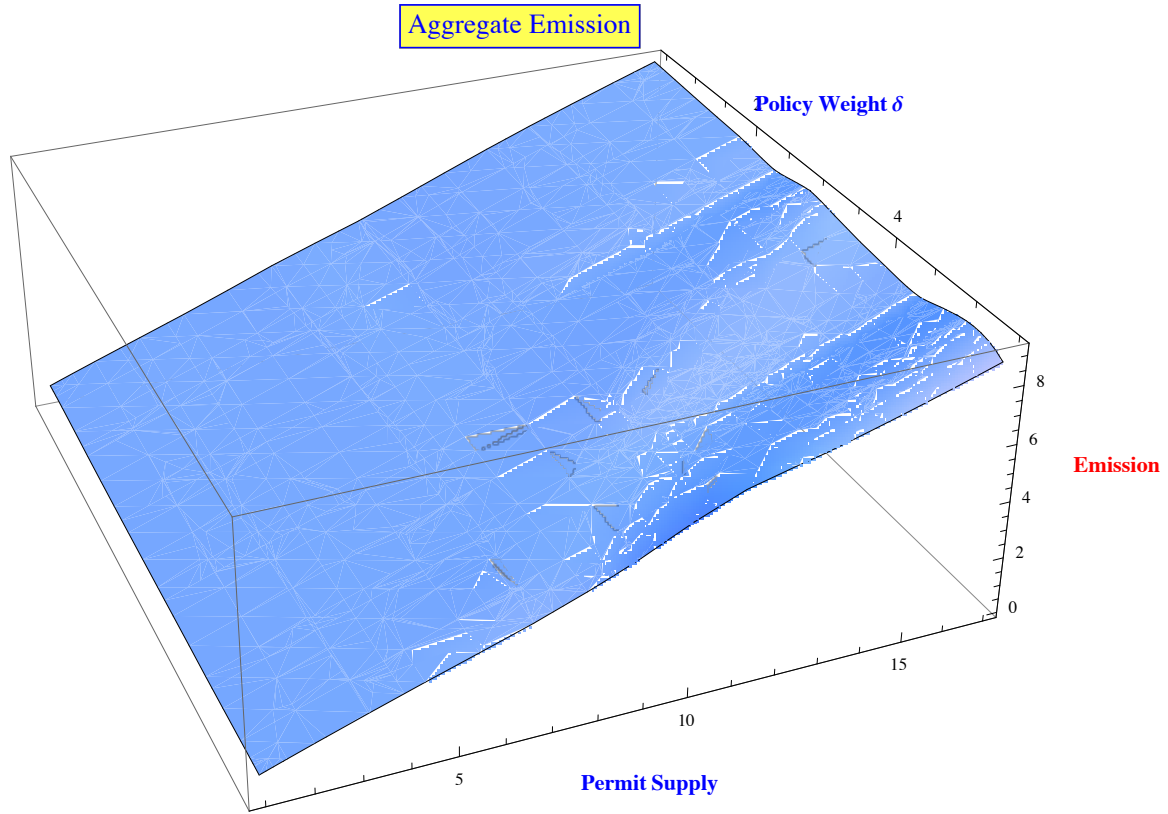The industry emission and production also seem (somewhat) monotonic along the changes:
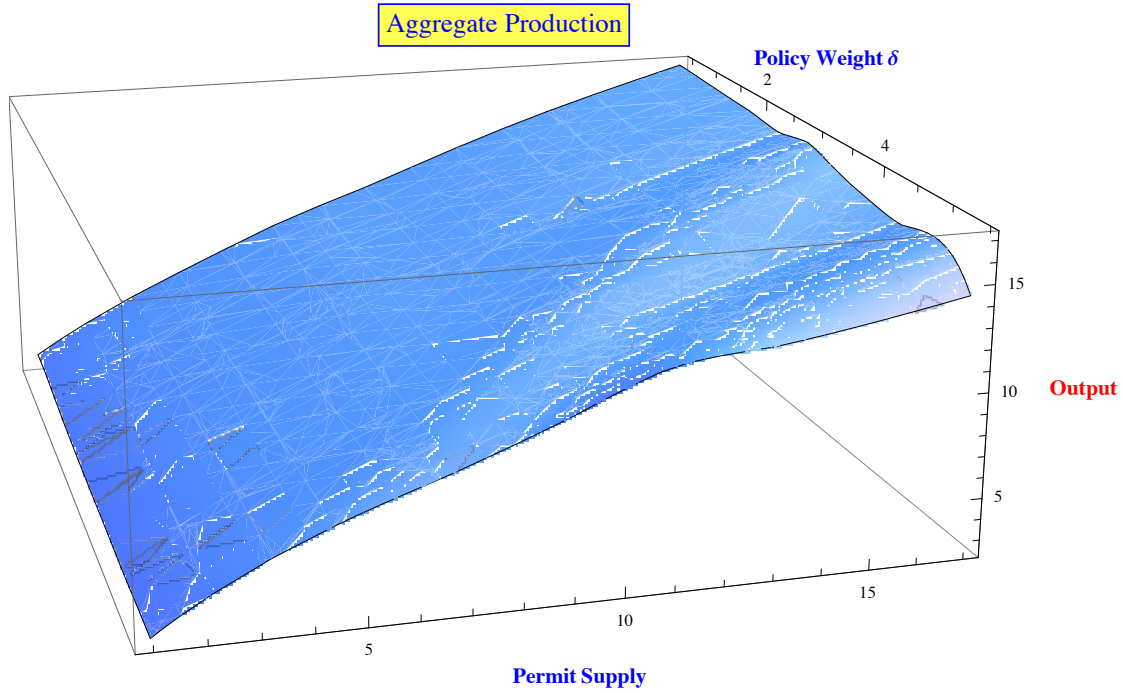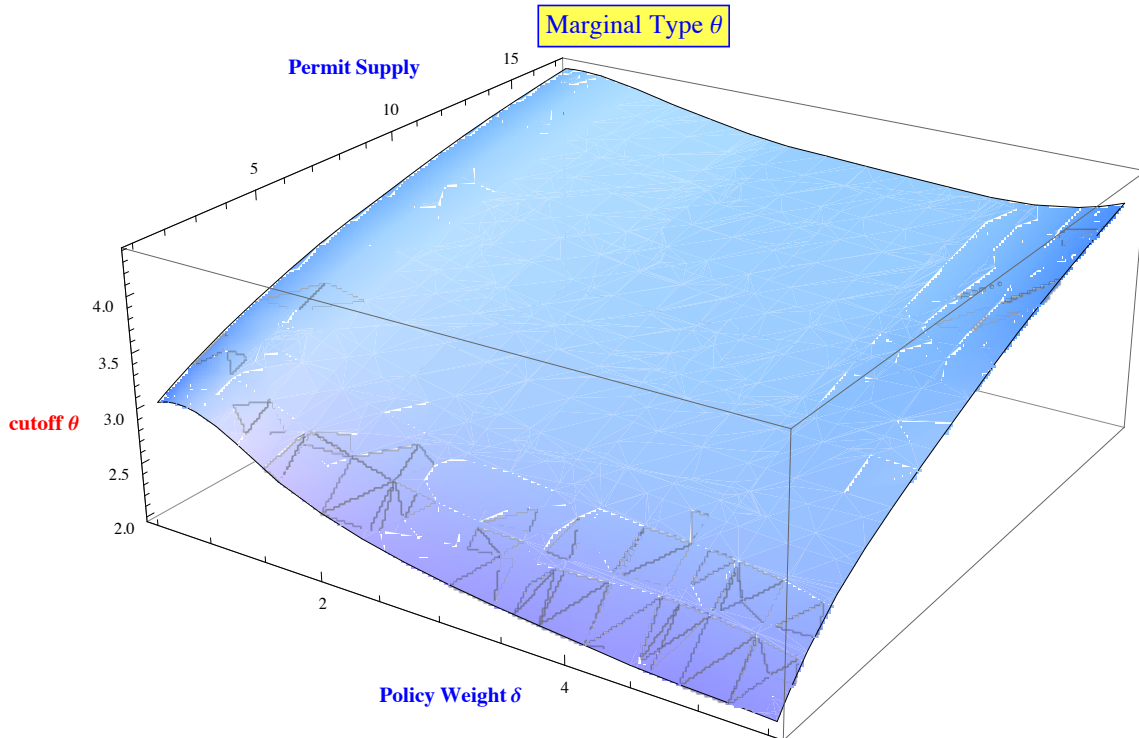
Figure 65.

Figure 66.

The marginal type of firm are not as smooth as the production and the pollution:

Figure 67.

The optimal policy and aggregate variables are summarized below:

Table 9. Optimal Variable Values under Log-normal Distribution						
Permit Supply	Policy Weight δ	Permit Price	Production	Emission	Marginal $\hat{\theta}$	Welfare
11.068	4.602	3.578	11.858	5.559	3.538	-0.0086

From the simulations, we find that the social welfare is not monotonically increasing in the scale of fine. In fact, there exists some value of the policy weight δ^* such that it maximize the welfare. This implies that, if the government has adopted the "maximum fine, minimum auditing" approach of enforcement, it needs to be careful for the steps it takes toward the "best fine," for at some point increasing fine will be detrimental to the economy, for raising fine induces firms leaving the industry, and hence too few are produce. Keep in mind that while adjusting the scale of penalty, the government should also set the number of permits accordingly, in order to alleviate the side effects of fluctuating permit price harming the industry and the economy.

CHAPTER 7: FINAL REMARKS

We have shown, in **Chapter 2** and **3**, that when there's uncertainty in the "observed emission," and if firms are risk averse, as the government adjusts its policy instruments, firms may act accordingly by adjusting their choice variables. That is, unlike in a world with certainty and/or with risk neutral firms, where the government's instruments are neutral to firms' behavior (i.e., as long as the expected fine remains unchanged, the government can arbitrarily change its monitoring frequency and penalty schedule), risk averse firms' optimal choices on production and its compliance decision varies as the government changes its policy.

A risk averse firm's behavior is less complicated when it faces an additive measurement error. As demonstrated in **Chapter 2**, a risk averse firm who exhibits CARA will increase its permit holding if the government increases the penalty schedule, cut the audit frequency while holding the expected fine unchanged. With multiplicative measurement error, however, a firm's optimal choices depends on its preferences. We get limited analytical results on firms' behavior in the permit markets, though: as equilibrium permit price increases, a risk averse firm with CARA, demand less permits.[98] With further assumptions on the utility functional form, we can show its emission and production also decrease as permit price rises. Changes in optimal decisions on pollution, production and compliances, with respect to the adjustment of policy weight (while holding expected fine fixed) and/or to the audit frequency, nevertheless, are not available to us analytically.

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[98] Though the intuition is that the results should hold for the results on other types of risk aversion, to make suck an assertion there need to be more rigorous analytical evidence.

In **Chapter 4**, we address the impacts of government's policy instruments on the market equilibrium. For a small open (in the product market) economy, on one hand, when the government increases its policy weight (again, while holding expected fine unchanged), the industry's size shrinks as less efficient firms leave the industry. On the other, when the government increases the number of permits, the industry can expand as lowered permit price entices new entrants in the market. Aggregate production and pollution both increase.

To get a clearer view on a firm's behavior when facing multiplicative measurement error, we use a numerical example to demonstrate the impacts of various parameters in **Chapter 5**. We first show the equilibrium under certainty. In the deterministic world, firms will demand the exact amount of permits as their emissions, since the government can adjust the fine/audit probability and the number of permits simultaneously to ensure compliance. We also note that, when there is some uncertainty such as the measurement error, however small, if the government raise the audit frequency close enough to resemble the deterministic world, the optimum will also converge to that of the certainty case.

Taking another step forward, when facing (significant) uncertainty (e.g. measurement error, $\lambda \sim U[0,2]$ in the model), firms buy the permits as hedges against being in "observed non-compliance", as in the additive case, and pollute less than the intended amount.

The impact of mean-preserving spread of the uncertainty is not monotonic in aggregate production and pollution, nonetheless. The industry grows in size, though, as the government issue more permits rising at optimum in response to the increasing risk.

Next, the robustness test shows that, under both the uniform and log-normal distributions for the error term, simulations on firms' behavior corroborate the analytical results that the degree of compliance increases as the government tightens its regulations (whether through increased audit frequency or aggravated penalty scale). As the policy weight increases, an individual firm's utility falls (consistent with *Proposition 3* in **Chapter 3**), while it produces and pollutes less; the industry dwindles as the marginal firm's type value decreases, and the aggregate production and emission decrease. The overall social welfare of the economy increases as the regulation tightens, for the cost of monitoring violations has not been imposed, and we have assumed there's no cost adjusting the weight on the penalty scale per se. Another interesting observation (in both the uniform and log-normal error cases) is that, more efficient firms, i.e. those with lower type values, have higher degree of compliance in terms of the permit-to-pollution ratio $\hat{\lambda}$, since they produce more and pollute more, and hence face higher risk of measurement error. Moreover, in contrast to the additive error case, in which firms' permit holding tends to go up as the government imposes more severe penalty schedule, when the measurement error is proportional to the emission, firms' permit demand actually falls (and thus the equilibrium permit price decreases) as the weight on the penalty increases (while a rise in the audit frequency alone will boost the permit demand). But as the regulations become more strict, firms respond by cutting back their production (and thus the pollution decreases). It can be inferred that, when the scale of penalty increases, firms react by cutting the production (and pollution) more drastically (since the permit demand is decreasing) than the decreased permit demand, such that the degree of compliance is increasing in the weight

on penalty (consistent with *Proposition 4* in **Chapter 3**), implying the production (and pollution) is more sensitive to the severity of penalty than the demand for permits.

Despite the model is sensitive to the parameterizations, the robustness test showed consistency of firms' behavior pattern and the industry's evolution regarding environmental regulations with that under the uniform distribution, suggesting that our results and conclusions are robust to different distributions of the measurement error.

However, more can be said on the model relaxations: when the weight on pollution damage (i.e., damage parameter α) increases, the optimal number of permits is expected to drop; when the agents become more risk averse (i.e., the CARA coefficient ϕ increases), they will be more conservative on the degree of compliance, though whether it is through production abatement or permit demand increase needs to be further checked.

APPENDIX

1. The Derivation of Proposition 1 in Chapter 5 (Footnote 72)

$$V_{ep} = 0 \cdot z - c_e \frac{\partial z}{\partial p} + \ell\pi \int_{\hat{\lambda}}^{\infty} u"(R_1)G'\lambda f(\lambda)d\lambda$$

$$= -c_e\left\{-\ell u"(R_0) - \ell\pi \int_{\hat{\lambda}}^{\infty}\left[u"(R_1)-u"(R_0)\right]f(\lambda)d\lambda\right\} + \ell\pi \int_{\hat{\lambda}}^{\infty} u"(R_1)G'\lambda f(\lambda)d\lambda$$

$$= -c_e\left\{\ell\phi u'(R_0) + \ell\phi\pi \int_{\hat{\lambda}}^{\infty}\left[u'(R_1)-u'(R_0)\right]f(\lambda)d\lambda\right\} - \ell\phi\pi \int_{\hat{\lambda}}^{\infty} u'(R_1)G'\lambda f(\lambda)d\lambda$$

$$= \ell\phi\left(-c_e z - \pi \int_{\hat{\lambda}}^{\infty} u(R_1)G'\lambda f(\lambda)d\lambda\right) = 0$$

The last equality is obtained by (3a): $V_e = -c_e z - \pi \int_{\hat{\lambda}}^{\infty} u'(R_1)G'\lambda f(\lambda)d\lambda = 0$.

2. The Derivation of the Corollary (Footnote 73)

Recall the risk averse utility functional form in (8): $u(R) = -\dfrac{\exp(-\phi R)}{\phi}$, $\phi > 0$, the FOC (3a)

can be rewritten as:

$$V_e \equiv \frac{\partial E(u)}{\partial e} = u'(R_0)\left\{-c_e\left[1 + \frac{\pi}{\delta} \int_{\lambda \geq \hat{\lambda}} \left(e^{\phi\delta G} - 1\right)f_\lambda d\lambda\right] - \pi \int_{\lambda \geq \hat{\lambda}} e^{\phi\delta G}G'\lambda f_\lambda d\lambda\right\} = 0,$$

and thus we have:

$$\pi\int_{\lambda \geq \hat{\lambda}} e^{\phi\delta G}\left(\delta G'\lambda + c_e\right)f_\lambda d\lambda = \left(-c_e\right)\left[\delta + \pi\left(1 - F(\hat{\lambda})\right)\right] > 0.$$

3. The Derivation of Utility Change in the Permit Price (Footnote 76)

To sign the direction of utility with respect to the permit price:

$$\frac{\partial V^*}{\partial p} = \left(\frac{\partial R_0}{\partial p}\right)\left\{u'(R_0) + \frac{\pi}{\delta}\int_{\hat{\lambda}}^{\infty}\left[u'(R_0 - \delta G) - u'(R_0)\right]f_\lambda d\lambda\right\} - \pi\int_{\hat{\lambda}}^{\infty} u'(R_0 - \delta G)G'\left(\lambda\frac{\partial e}{\partial p} - \frac{\partial \ell}{\partial p}\right)f_\lambda d\lambda$$

.

Denote $z^* = u'(R_0^*) + \dfrac{\pi}{\delta}\int_{\hat{\lambda}}^{\infty}\left[u'(R_0^* - \delta G) - u'(R_0^*)\right]f(\lambda)d\lambda$. From the firm's FOC:

$$\pi\int_{\hat{\lambda}}^{\infty}u'(R_0 - \delta G)G'\lambda f_\lambda\, d\lambda = -c_e z \quad, \quad \pi\int_{\hat{\lambda}}^{\infty}u'(R_0 - \delta G)G'f_\lambda\, d\lambda = pz \qquad , \qquad \text{and}$$

$\dfrac{\partial R_0}{\partial p} = -c_e\dfrac{\partial e}{\partial p} - p\dfrac{\partial \ell}{\partial p} - \ell$, then $\dfrac{\partial V^*}{\partial p} = \left(\dfrac{\partial R_0}{\partial p}\right)z + c_e z\left(\dfrac{\partial e}{\partial p}\right) + pz\left(\dfrac{\partial \ell}{\partial p}\right) = -\ell z < 0$.

4. Robustness Test on α

According to the social welfare function:

$$\max_{\{\pi,\delta,L\}} SW = N\left(1 - H(\hat{\theta})\right)u(w + T - \alpha E^2) + N\int_{\underline{\theta}}^{\hat{\theta}}\mathop{E}_{\lambda}\left[u(R(\theta))\right]h(\theta)d\theta$$

$$= \exp\left(-\phi(T - \alpha E^2)\right)\left\{-N\left(1 - H(\hat{\theta})\right)\dfrac{\exp(-\phi w)}{\phi} + N\int_{\underline{\theta}}^{\hat{\theta}}V^*(\theta)\cdot h(\theta)d\theta\right\},$$

when the damage coefficient α increases, social welfare decreases ceteris paribus.

In a deterministic world, when α goes from 0.1 to 0.8, the optimal permit supply drops drastically to merely ≈ 1.113. The following graphs depict the optimal number of permits and social welfare's paths along the increasing α:
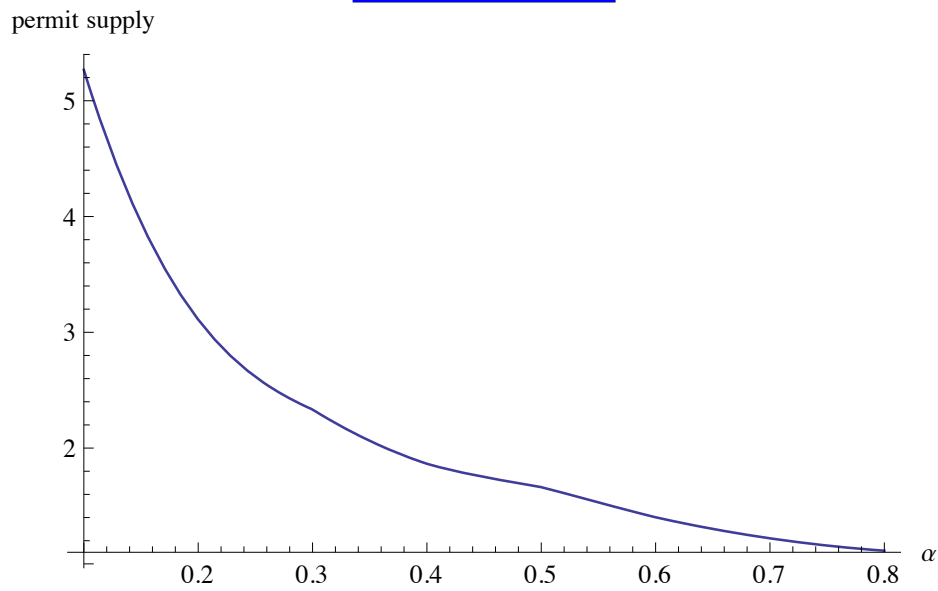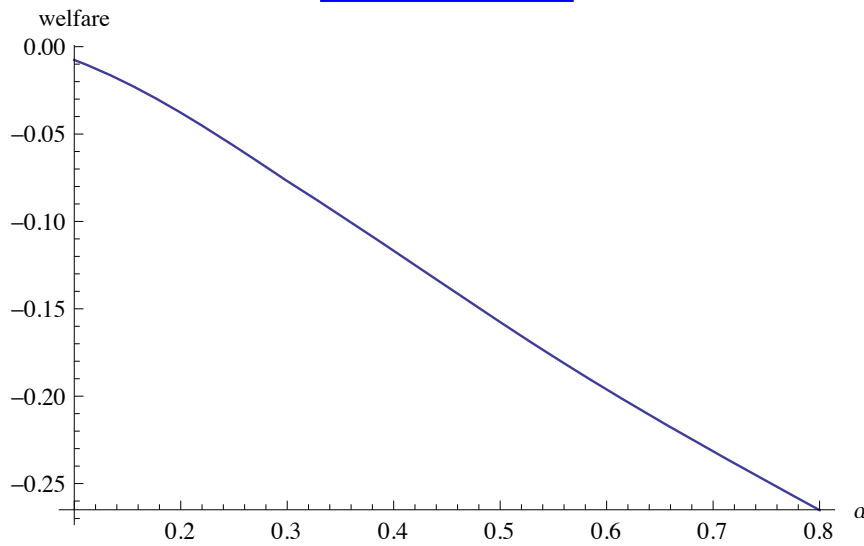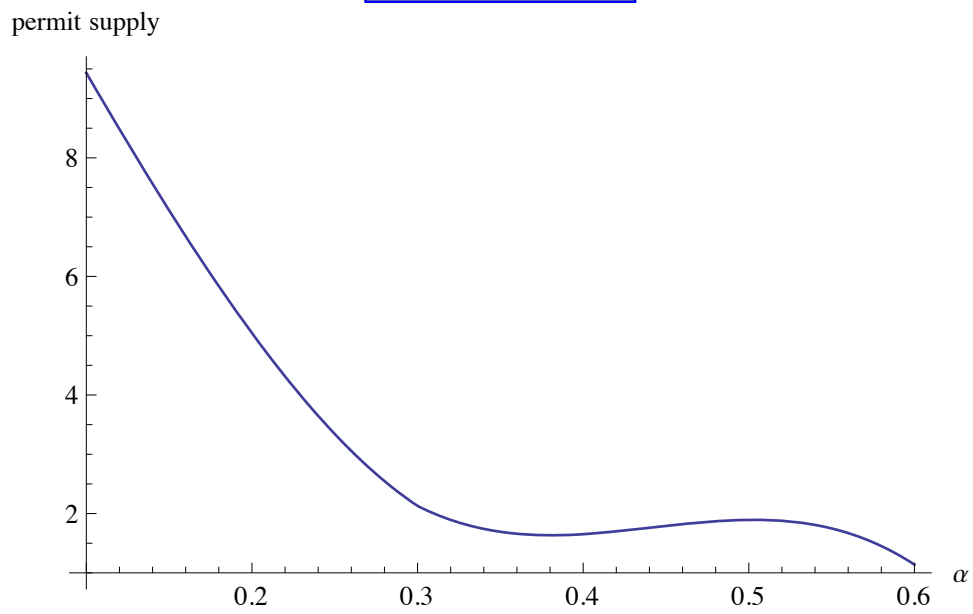
Figure A1. Optimal Permit Supply Under Certainty

Figure A2. Social Welfare Under Certainty

The tendency of decreasing welfare and optimal permit supply remains unchanged when we introduces uncertainty (with the error $\lambda \sim U[0,2]$), as illustrated in the following graphs:

Figure A3. Optimal Permit Supply When $\lambda \sim U[0,2]$

Permit Supply

permit supply

Figure A4. Social Welfare When $\lambda \sim U[0, 2]$

Social Welfare

welfare
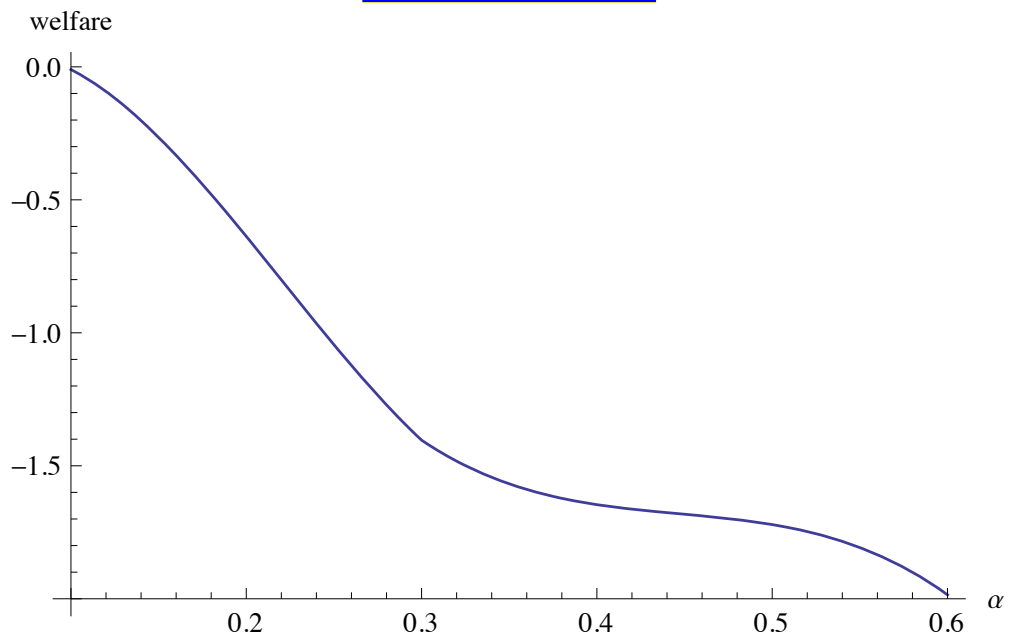
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